1–1. Represent each of the following quantities with combinations of units in the correct SI form, using an appropriate prefix: (a) $\mathrm{mm \cdot MN}$, (b) $\mathrm{Mg/mm}$, (c) $\mathrm{km/ms}$, (d) $\mathrm{kN/(mm)^2}$.

SOLUTION

a) $\mathrm{mm \cdot MN} = (10^{-3}\,\mathrm{m})(10^{6}\,\mathrm{N}) = 10^{3}\,\mathrm{N \cdot m} = \mathrm{kN \cdot m}$ **Ans.**

b) $\mathrm{Mg/mm} = (10^{6}\,\mathrm{g})/(10^{-3}\,\mathrm{m}) = 10^{9}\,\mathrm{g/m} = \mathrm{Gg/m}$ **Ans.**

c) $\mathrm{km/ms} = (10^{3}\,\mathrm{m})/(10^{-3}\,\mathrm{s}) = 10^{6}\,\mathrm{m/s} = \mathrm{Mm/s}$ **Ans.**

d) $\mathrm{kN/(mm)^2} = (10^{3}\,\mathrm{N})/(10^{-3}\,\mathrm{m})^2 = 10^{9}\,\mathrm{N/m^2} = \mathrm{GN/m^2}$ **Ans.**

Ans:
a) $\mathrm{kN \cdot m}$
b) $\mathrm{Gg/m}$
c) $\mathrm{Mm/s}$
d) $\mathrm{GN/m^2}$

1–2. Evaluate each of the following to three significant figures, and express each answer in SI units using an appropriate prefix: (a) $[4.86(10^6)]^2$ mm, (b) $(348\text{ mm})^3$, (c) $(83\,700\text{ mN})^2$.

SOLUTION

a) $[4.86(10^6)]^2\,\text{mm} = [4.86(10^6)]^2(10^{-3}\,\text{m}) = 23.62(10^9)\text{ m} = 23.6\text{ Gm}$ **Ans.**

b) $(348\text{ mm})^3 = [348(10^{-3})\text{ m}]^3 = 42.14(10^{-3})\text{ m}^3 = 42.1(10^{-3})\text{ m}^3$ **Ans.**

c) $(83{,}700\text{ mN})^2 = [83{,}700(10^{-3})\text{ N}]^2 = 7.006(10^3)\text{ N}^2 = 7.01(10^3)\text{ N}^2$ **Ans.**

Ans:
a) 23.6 Gm
b) $42.1(10^{-3})\text{ m}^3$
c) $7.01(10^3)\text{ N}^2$

1–3. Evaluate each of the following to three significant figures, and express each answer in SI units using an appropriate prefix: (a) 749 μm/63 ms, (b) (34 mm)(0.0763 Ms)/263 mg, (c) (4.78 mm)(263 Mg).

SOLUTION

a) $749\ \mu\text{m}/63\ \text{ms} = 749\left(10^{-6}\right)\text{m}/63\left(10^{-3}\right)\text{s} = 11.88\left(10^{-3}\right)\text{m/s}$
$= 11.9\ \text{mm/s}$ **Ans.**

b) $(34\ \text{mm})(0.0763\ \text{Ms})/263\ \text{mg} = \left[34\left(10^{-3}\right)\text{m}\right]\left[0.0763\left(10^{6}\right)\text{s}\right]/\left[263\left(10^{-6}\right)\left(10^{3}\right)\text{g}\right]$
$= 9.86\left(10^{6}\right)\text{m}\cdot\text{s/kg} = 9.86\ \text{Mm}\cdot\text{s/kg}$ **Ans.**

c) $(4.78\ \text{mm})(263\ \text{Mg}) = \left[4.78\left(10^{-3}\right)\text{m}\right]\left[263\left(10^{6}\right)\text{g}\right]$
$= 1.257\left(10^{6}\right)\text{g}\cdot\text{m} = 1.26\ \text{Mg}\cdot\text{m}$ **Ans.**

Ans:
a) 11.9 mm/**s**
b) 9.86 Mm·s/kg
c) 1.26 Mg·m

***1–4.** Convert the following temperatures: (a) 250 K to degrees Celsius, (b) 322°F to degrees Rankine, (c) 230°F to degrees Celsius, (d) 40°C to degrees Fahrenheit.

SOLUTION

a) $T_K = T_C + 273; \quad 250\text{ K} = T_C + 273 \quad T_C = -23.0°\text{C}$ **Ans.**

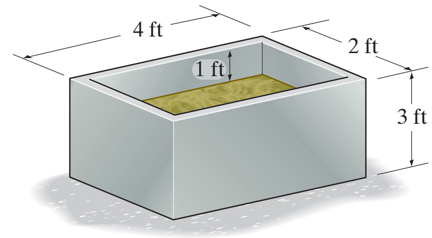
b) $T_R = T_F + 460 = 322°\text{F} + 460 = 782°\text{R}$ **Ans.**

c) $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(230°\text{F} - 32) = 110°\text{C}$ **Ans.**

d) $T_C = \frac{5}{9}(T_F - 32); \quad 40°\text{C} = \frac{5}{9}(T_F - 32) \quad T_F = 104°\text{F}$ **Ans.**

Ans:
a) $-23.0°\text{C}$
b) $782°\text{R}$
c) $110°\text{C}$
d) $104°\text{F}$

1–5. The tank contains a liquid having a density of 1.22 slug/ft^3. Determine the weight of the liquid when it is at the level shown.



SOLUTION

The specific weight of the liquid and the volume of the liquid are

$$\gamma = \rho g = \left(1.22 \text{ slug/ft}^3\right)\left(32.2 \text{ ft/s}^2\right) = 39.284 \text{ lb/ft}^3$$

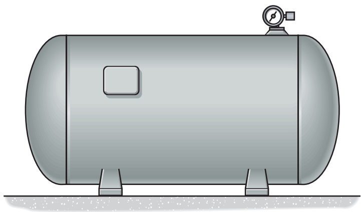
$$\forall = (4 \text{ ft})(2 \text{ ft})(2 \text{ ft}) = 16 \text{ ft}^3$$

Then the weight of the liquid is

$$W = \gamma \forall = \left(39.284 \text{ lb/ft}^3\right)\left(16 \text{ ft}^3\right) = 628.54 \text{ lb} = 629 \text{ lb}$$ **Ans.**

Ans:
$W = 629$ lb

1–6. If air within the tank is at an absolute pressure of 680 kPa and a temperature of 70°C, determine the weight of the air inside the tank. The tank has an interior volume of $1.35\ \text{m}^3$.



SOLUTION

From the table in Appendix A, the gas constant for air is $R = 286.9\ \text{J/kg}\cdot\text{K}$.

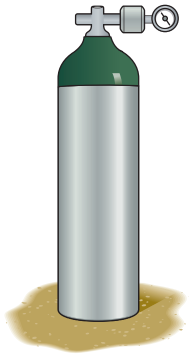
$$p = \rho R T$$
$$680(10^3)\ \text{N/m}^2 = \rho(286.9\ \text{J/kg}\cdot\text{K})(70° + 273)\ \text{K}$$
$$\rho = 6.910\ \text{kg/m}^3$$

The weight of the air in the tank is

$$W = \rho g V\!\!\!/ = (6.910\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.35\ \text{m}^3)$$
$$= 91.5\ \text{N}$$ **Ans.**

Ans:
$W = 91.5\ \text{N}$

1–7. The bottle tank has a volume of $0.35\ \text{m}^3$ and contains 40 kg of nitrogen at a temperature of 40°C. Determine the absolute pressure in the tank.



SOLUTION

The density of nitrogen in the tank is

$$\rho = \frac{m}{\forall} = \frac{40\ \text{kg}}{0.35\ \text{m}^3} = 114.29\ \text{kg/m}^3$$

From the table in Appendix A, the gas constant for nitrogen is $R = 296.8\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law,

$$p = \rho R T$$

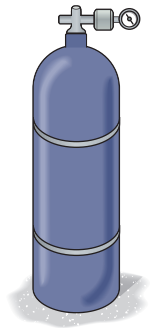
$$p = \left(114.29\ \text{kg/m}^3\right)(296.8\ \text{J/kg}\cdot\text{K})(40°\text{C} + 273)\ \text{K}$$

$$= 10.62\left(10^6\right)\text{Pa}$$

$$= 10.6\ \text{MPa} \qquad \textbf{Ans.}$$

Ans:
$p = 10.6\ \text{MPa}$

***1–8.** The bottle tank contains nitrogen having a temperature of 60°C. Plot the variation of the pressure in the tank (vertical axis) versus the density for $0 \leq \rho \leq 5\ \mathrm{kg/m^3}$. Report values in increments of $\Delta p = 50\ \mathrm{kPa}$.



SOLUTION

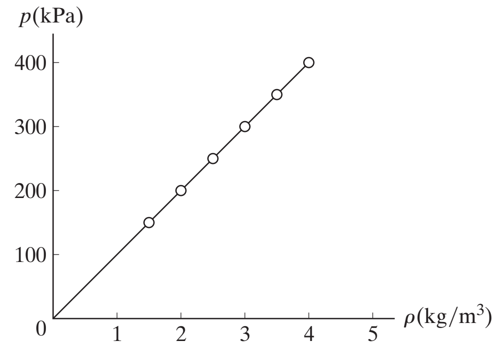
From the table in Appendix A, the gas constant for nitrogen is $R = 296.8\ \mathrm{J/kg \cdot K}$. The constant temperature is $T = (60°\mathrm{C} + 273)\ \mathrm{K} = 333\ \mathrm{K}$. Applying the ideal gas law,

$$\begin{aligned} p &= \rho R T \\ p &= \rho(296.8\ \mathrm{J/kg \cdot K})(333\ \mathrm{K}) \\ p &= (98{,}834\rho)\ \mathrm{Pa} \\ &= (98.8\rho)\ \mathrm{kPa} \end{aligned}$$

Ans.

p (kPa)	150	200	250	300	350	400
ρ (kg/m^3)	1.52	2.02	2.53	3.04	3.54	4.05

The plot of p vs ρ is shown in Fig. a.



(a)

Ans:
$p = (98.8\rho)\ \mathrm{kPa}$

1–9. Determine the specific weight of hydrogen when the temperature is 85°C and the absolute pressure is 4 MPa.

SOLUTION

From the table in Appendix A, the gas constant for hydrogen is $R = 4124\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law,

$$p = \rho RT$$
$$4\left(10^6\right)\ \text{N/m}^2 = \rho(4124\ \text{J/kg}\cdot\text{K})(85°\text{C} + 273)\ \text{K}$$
$$\rho = 2.7093\ \text{kg/m}^3$$

Then the specific weight of hydrogen is

$$\begin{aligned}\gamma = \rho g &= \left(2.7093\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)\\ &= 26.58\ \text{N/m}^3\\ &= 26.6\ \text{N/m}^3\end{aligned}$$ **Ans.**

Ans:
$\gamma = 26.6\ \text{N/m}^3$

1–10. Dry air at 25°C has a density of 1.23 kg/m^3. But if it has 100% humidity at the same pressure, its density is 0.65% less. At what temperature would dry air produce this same smaller density?

SOLUTION

For both cases, the pressures are the same. Applying the ideal gas law with $\rho_1 = 1.23\ \text{kg/m}^3$, $\rho_2 = \left(1.23\ \text{kg/m}^3\right)(1 - 0.0065) = 1.222005\ \text{kg/m}^3$ and $T_1 = (25°\text{C} + 273) = 298\ \text{K}$,

$$p = \rho_1 R T_1 = \left(1.23\ \text{kg/m}^3\right) R (298\ \text{K}) = 366.54\ \text{R}$$

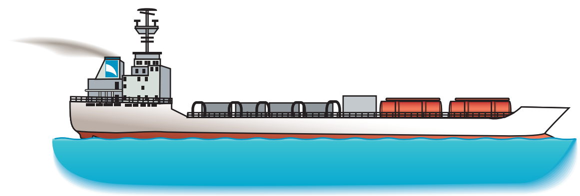
Then

$$p = \rho_2 R T_2; \quad 366.54\ R = \left(1.222005\ \text{kg/m}^3\right) R (T_C + 273)$$

$$T_C = 26.9°\text{C} \qquad \textbf{Ans.}$$

Ans:
$T_C = 26.9°\text{C}$

1–11. The tanker carries $900(10^3)$ barrels of crude oil in its hold. Determine the weight of the oil if its specific gravity is 0.940. Each barrel contains 42 gallons, and there are 7.48 gal/ft^3.



SOLUTION

The specific weight of the crude oil is

$$\gamma_o = S_o\gamma_w = 0.940\left(62.4\ \text{lb}/\text{ft}^3\right) = 58.656\ \text{lb}/\text{ft}^3$$

The volume of the crude oil is

$$\mathcal{V}_o = \left[900(10^3)\ \cancel{\text{bl}}\right]\left(\frac{42\ \cancel{\text{gal}}}{1\ \cancel{\text{bl}}}\right)\left(\frac{1\ \text{ft}^3}{7.48\ \cancel{\text{gal}}}\right) = 5.0535(10^6)\ \text{ft}^3$$

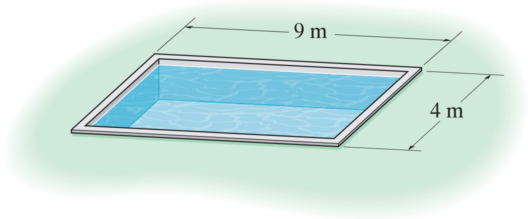
Then, the weight of the crude oil is

$$\begin{aligned} W_o &= \gamma_o\mathcal{V}_o = \left(58.656\ \text{lb}/\text{ft}^3\right)\left[5.0535\left(10^6\right)\ \text{ft}^3\right] \\ &= 296.41\left(10^6\right)\ \text{lb} \\ &= 296\left(10^6\right)\ \text{lb} \end{aligned}$$

Ans.

Ans:
$W_o = 296\left(10^6\right)$ lb

***1–12.** Water in the swimming pool has a measured depth of 3.03 m when the temperature is 5°C. Determine its approximate depth when the temperature becomes 35°C. Neglect losses due to evaporation.



SOLUTION

From Appendix A, at $T_1 = 5°\text{C}$, $(\rho_w)_1 = 1000.0\ \text{kg/m}^3$. The volume of the water is $\mathcal{V} = Ah$. Thus, $\mathcal{V}_1 = (9\ \text{m})(4\ \text{m})(3.03\ \text{m})$. Then

$$(\rho_w)_1 = \frac{m}{\mathcal{V}_1}; \qquad 1000.0\ \text{kg/m}^3 = \frac{m}{36\ \text{m}^2(3.03\ \text{m})}$$

$$m = 109.08\left(10^3\right)\ \text{kg}$$

At $T_2 = 35°\text{C}$, $(\rho_w)_2 = 994.0\ \text{kg/m}^3$. Then

$$(\rho_w)_2 = \frac{m}{\mathcal{V}_2}; \qquad 994.0\ \text{kg/m}^3 = \frac{109.08\left(10^3\right)}{\left(36\ \text{m}^2\right)h}$$

$$h = 3.048\ \text{m} = 3.05\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$h = 3.05\ \text{m}$

1–13. Determine the weight of carbon tetrachloride that should be mixed with 15 lb of glycerin so that the combined mixture has a density of 2.85 slug/ft^3.

SOLUTION

From the table in Appendix A, the densities of glycerin and carbon tetrachloride at s.t.p. are $\rho_g = 2.44\ \text{slug/ft}^3$ and $\rho_{ct} = 3.09\ \text{slug/ft}^3$, respectively. Thus, their volumes are given by

$$\rho_g = \frac{m_g}{V_g}; \quad 2.44\ \text{slug/ft}^3 = \frac{(15\ \text{lb})/(32.2\ \text{ft/s}^2)}{V_g} \qquad V_g = 0.1909\ \text{ft}^3$$

$$\rho_{ct} = \frac{m_{ct}}{V_{ct}}; \quad 3.09\ \text{slug/ft}^3 = \frac{W_{ct}/(32.2\ \text{ft/s}^2)}{V_{ct}} \qquad V_{ct} = (0.01005 W_{ct})\ \text{ft}^3$$

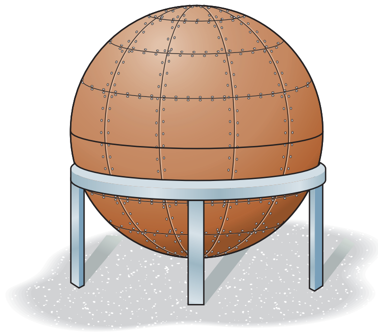
The density of the mixture is

$$\rho_m = \frac{m_m}{V_m}; \quad 2.85\ \text{slug/ft}^3 = \frac{W_{ct}/(32.2\ \text{ft/s}^2) + (15\ \text{lb})/(32.2\ \text{ft/s}^2)}{0.1909\ \text{ft}^3 + 0.01005 W_{ct}}$$

$$W_{ct} = 32.5\ \text{lb}$$ **Ans.**

Ans:
$W_{ct} = 32.5\ \text{lb}$

1–14. The tank contains air at a temperature of 18°C and an absolute pressure of 160 kPa. If the volume of the tank is 3.48 m^3 and the temperature rises to 42°C, determine the mass of air that must be removed from the tank to maintain the same pressure.



SOLUTION

For $T_1 = (18°\text{C} + 273)\text{ K} = 291\text{ K}$ and $R = 286.9\text{ J/kg}\cdot\text{K}$ for air (Appendix A), the ideal gas law gives

$$p_1 = \rho_1 R T_1; \qquad 160(10^3)\text{ N/m}^2 = \rho_1(286.9\text{ J/kg}\cdot\text{K})(291\text{ K})$$

$$\rho_1 = 1.9164\text{ kg/m}^3$$

Thus, the mass of the air at T_1 is

$$m_1 = \rho_1 \forall = (1.9164\text{ kg/m}^3)\,(3.48\text{ m}^3) = 6.6692\text{ kg}$$

For $T_2 = (42°\text{C} + 273)\text{ K} = 315\text{ K}$, and $R = 286.9\text{ J/kg}\cdot\text{K}$,

$$p_2 = \rho_2 R T_2; \qquad 160(10^3)\text{ N/m}^2 = \rho_2(286.9\text{ J/kg}\cdot\text{K})(315\text{ K})$$

$$\rho_2 = 1.7704\text{ kg/m}^3$$

Thus, the mass of air at T_2 is

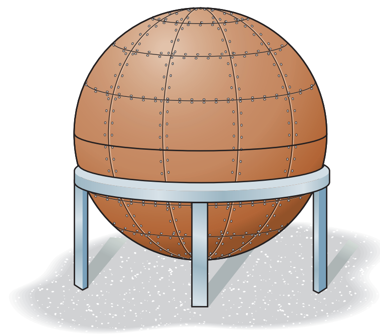
$$m_2 = \rho_2 \forall = (1.7704\text{ kg/m}^3)\,(3.48\text{ m}^3) = 6.1611\text{ kg}$$

Finally, the mass of air that must be removed is

$$\Delta m = m_1 - m_2 = 6.6692\text{ kg} - 6.1611\text{ kg} = 0.508\text{ kg} \qquad \textbf{Ans.}$$

Ans:
$\Delta m = 0.508$ kg

1–15. The tank contains 4 kg of air at an absolute pressure of 350 kPa and a temperature of 18°C. If 0.8 kg of air is added to the tank and the temperature rises to 38°C, determine the resulting pressure in the tank.



SOLUTION

For $T_1 = (18°\text{C} + 273)\text{ K} = 291\text{ K}$, $p_1 = 350\text{ kPa}$ and $R = 286.9\text{ J/kg}\cdot\text{K}$ for air (Appendix A), the ideal gas law gives

$$p_1 = \rho_1 R T_1; \quad 350(10^3)\text{ N/m}^2 = \rho_1(286.9\text{ J/kg}\cdot\text{K})(291\text{ K})$$
$$\rho_1 = 4.1922\text{ kg/m}^3$$

Since the volume is constant,

$$\mathcal{V} = \frac{m_1}{\rho_1} = \frac{m_2}{\rho_2}; \quad \rho_2 = \frac{m_2}{m_1}\rho_1$$

Here, $m_1 = 4\text{ kg}$ and $m_2 = (4 + 0.8)\text{ kg} = 4.8\text{ kg}$

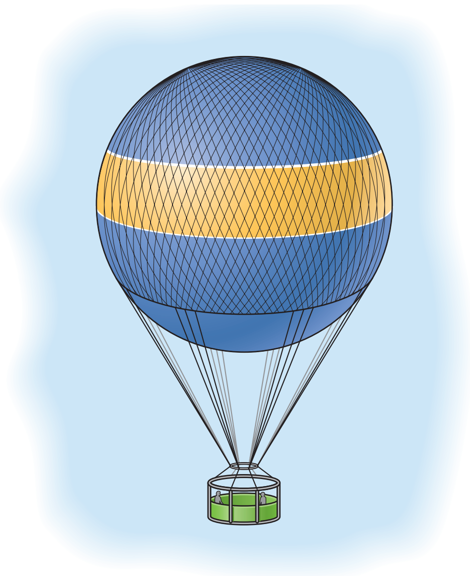
$$\rho_2 = \left(\frac{4.8\text{ kg}}{4\text{ kg}}\right)(4.1922\text{ kg/m}^3) = 5.0307\text{ kg/m}^3$$

Again, applying the ideal gas law with $T_2 = (38°\text{C} + 273)\text{ K} = 311\text{ K}$,

$$p_2 = \rho_2 R T_2; = (5.0307\text{ kg/m}^3)(286.9\text{ J/kg}\cdot\text{K})(311\text{ K})$$
$$= 448.86(10^3)\text{ Pa}$$
$$= 449\text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$p_2 = 449\text{ kPa}$

***1–16.** The 8-m-diameter spherical balloon is filled with helium that is at a temperature of 28°C and an absolute pressure of 106 kPa. Determine the weight of the helium contained in the balloon. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.



SOLUTION

For helium, the gas constant is $R = 2077\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law at $T = (28 + 273)\ \text{K} = 301\ \text{K}$,

$$p = \rho RT; \qquad 106(10^3)\ \text{N/m}^2 = \rho(2077\ \text{J/kg}\cdot\text{K})(301\ \text{K})$$
$$\rho = 0.1696\ \text{kg/m}^3$$

Here,

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4\ \text{m})^3 = \frac{256}{3}\pi\ \text{m}^3$$

Then, the mass of the helium is

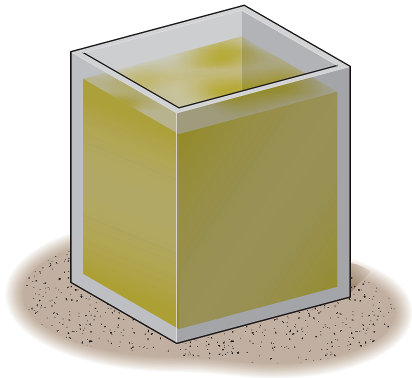
$$M = \rho V = \left(0.1696\ \text{kg/m}^3\right)\left(\frac{256}{3}\pi\ \text{m}^3\right) = 45.45\ \text{kg}$$

Thus,

$$W = mg = \left(45.45\ \text{kg}\right)\left(9.81\ \text{m/s}^2\right) = 445.90\ \text{N} = 446\ \text{N} \qquad \textbf{Ans.}$$

Ans:
$W = 446\ \text{N}$

1–17. Gasoline is mixed with 8 ft^3 of kerosene so that the volume of the mixture in the tank becomes 12 ft^3. Determine the specific weight and the specific gravity of the mixture at standard temperature and pressure.



SOLUTION

From the table in Appendix A, the densities of gasoline and kerosene at s.t.p. are $\rho_g = 1.41\ \text{slug/ft}^3$ and $\rho_k = 1.58\ \text{slug/ft}^3$, respectively. The volume of gasoline is

$$\forall_g = 12\ \text{ft}^3 - 8\ \text{ft}^3 = 4\ \text{ft}^3$$

Then the total weight of the mixture is therefore

$$\begin{aligned} W_m &= \rho_g g \forall_g + \rho_k g \forall_k \\ &= \left(1.41\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left(4\ \text{ft}^2\right) + \left(1.58\ \text{slug/ft}^3\right)\left(32.2\ \text{ft/s}^2\right)\left(8\ \text{ft}^3\right) \\ &= 588.62\ \text{lb} \end{aligned}$$

Thus, the specific weight and specific gravity of the mixture are

$$\gamma_m = \frac{W_m}{\forall_m} = \frac{588.62\ \text{lb}}{12\ \text{ft}^3} = 49.05\ \text{lb/ft}^3 = 49.1\ \text{lb/ft}^3 \qquad \textbf{Ans.}$$

$$S_m = \frac{\gamma_m}{\gamma_w} = \frac{49.05\ \text{lb/ft}^3}{62.4\ \text{lb/ft}^3} = 0.786 \qquad \textbf{Ans.}$$

Ans:
$\gamma_m = 49.1\ \text{lb/ft}^3$
$S_m = 0.786$

1–18. Determine the change in the density of oxygen when the absolute pressure changes from 345 kPa to 286 kPa, while the temperature *remains constant* at 25°C. This is called an *isothermal process*.

SOLUTION

Applying the ideal gas law with $T_1 = (25°\text{C} + 273)\text{ K} = 298\text{ K}$, $p_1 = 345\text{ kPa}$ and $R = 259.8\text{ J/kg}\cdot\text{K}$ for oxygen (table in Appendix A),

$$p_1 = \rho_1 R T_1; \qquad 345(10^3)\text{ N/m}^2 = \rho_1(259.8\text{ J/kg}\cdot\text{K})(298\text{ K})$$
$$\rho_1 = 4.4562\text{ kg/m}^3$$

For $p_2 = 286\text{ kPa}$ and $T_2 = T_1 = 298\text{ K}$,

$$p_2 = \rho_2 R T_2; \qquad 286(10^3)\text{ N/m}^2 = \rho_2(259.8\text{ J/kg}\cdot\text{K})(298\text{ K})$$
$$\rho_2 = 3.6941\text{ kg/m}^3$$

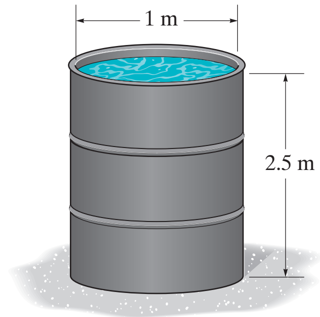
Thus, the change in density is

$$\Delta\rho = \rho_2 - \rho_1 = 3.6941\text{ kg/m}^3 - 4.4562\text{ kg/m}^3$$
$$= -0.7621\text{ kg/m}^3 = -0.762\text{ kg/m}^3 \qquad \textbf{Ans.}$$

The negative sign indicates a decrease in density.

Ans:
$\Delta\rho = -0.762\text{ kg/m}^3$

1–19. The container is filled with water at a temperature of 25°C and a depth of 2.5 m. If the container has a mass of 30 kg, determine the combined weight of the container and the water.



SOLUTION

From Appendix A, $\rho_w = 997.1\ \mathrm{kg/m^3}$ at $T = 25°\mathrm{C}$. Here, the volume of water is

$$\forall = \pi r^2 h = \pi(0.5\ \mathrm{m})^2(2.5\ \mathrm{m}) = 0.625\pi\ \mathrm{m}^3$$

Thus, the mass of water is

$$M_w = \rho_w \forall = 997.1\ \mathrm{kg/m^3}\left(0.625\pi\ \mathrm{m^3}\right) = 1957.80\ \mathrm{kg}$$

The total mass is

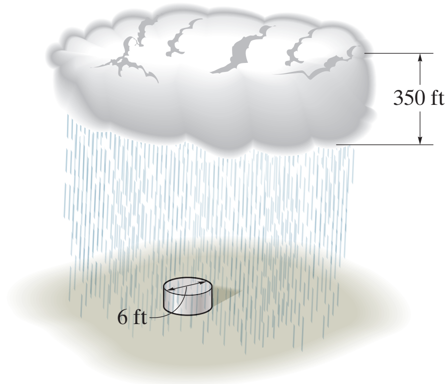
$$M_T = M_w + M_c = (1957.80 + 30)\ \mathrm{kg} = 1987.80\ \mathrm{kg}$$

Then the total weight is

$$W = M_T g = (1987.80\ \mathrm{kg})\left(9.81\ \mathrm{m/s^2}\right) = 19\,500\ \mathrm{N} = 19.5\ \mathrm{kN} \qquad \textbf{Ans.}$$

Ans:
$W = 19.5\ \mathrm{kN}$

***1–20.** The rain cloud has an approximate volume of 6.50 mile^3 and an average height, top to bottom, of 350 ft. If a cylindrical container 6 ft in diameter collects 2 in. of water after the rain falls out of the cloud, estimate the total weight of rain that fell from the cloud. 1 mile = 5280 ft.



SOLUTION

The volume of rain water collected is $\mathcal{V}_w = \pi(3\text{ ft})^2\left(\frac{2}{12}\text{ ft}\right) = 1.5\pi\text{ ft}^3$. Then, the weight of the rain water is $W_w = \gamma_w \mathcal{V}_w = \left(62.4\text{ lb/ft}^3\right)\left(1.5\pi\text{ ft}^3\right) = 93.6\pi\text{ lb}$. Here, the volume of the overhead cloud that produced this amount of rain is

$$\mathcal{V}_c' = \pi(3\text{ ft})^2(350\text{ ft}) = 3150\pi\text{ ft}^3$$

Thus,

$$\gamma_c = \frac{W}{\mathcal{V}_c'} = \frac{93.6\pi\text{ lb}}{3150\pi\text{ ft}^3} = 0.02971\text{ lb/ft}^3$$

Then

$$W_c = \gamma_c \mathcal{V}_c = \left(0.02971\,\frac{\text{lb}}{\text{ft}^3}\right)\left[(6.50)\left(\frac{5280^3\text{ ft}^3}{1}\right)\right]$$

$$= 28.4\left(10^9\right)\text{ lb} \qquad \textbf{Ans.}$$

Ans:
$W_c = 28.4\left(10^9\right)\text{ lb}$

1–21. A volume of 8 m^3 of oxygen initially at 80 kPa of absolute pressure and 15°C is subjected to an absolute pressure of 25 kPa while the temperature remains constant. Determine the new density and volume of the oxygen.

SOLUTION

From the table in Appendix A, the gas constant for oxygen is $R = 259.8\ \text{J/kg}\cdot\text{K}$. Applying the ideal gas law,

$$p_1 = \rho_1 R T_1; \quad 80(10^3)\ \text{N/m}^3 = \rho_1(259.8\ \text{J/kg}\cdot\text{K})(15^\circ\text{C} + 273)\ \text{K}$$
$$\rho_1 = 1.0692\ \text{kg/m}^3$$

For $T_2 = T_1$ and $p_2 = 25$ kPa,

$$\frac{p_1}{p_2} = \frac{\rho_1 R T_1}{\rho_2 R T_2}; \quad \frac{p_1}{p_2} = \frac{\rho_1}{\rho_2}$$
$$\frac{80\ \text{kPa}}{25\ \text{kPa}} = \frac{1.0692\ \text{kg/m}^3}{\rho_2}$$
$$\rho_2 = 0.3341\ \text{kg/m}^3 = 0.334\ \text{kg/m}^3 \qquad \textbf{Ans.}$$

The mass of the oxygen is

$$m = \rho_1 V_1 = (1.0692\ \text{kg/m}^3)(8\ \text{m}^3) = 8.5536\ \text{kg}$$

Since the mass of the oxygen is constant regardless of the temperature and pressure,

$$m = \rho_2 V_2; 8.5536\ \text{kg} = (0.3341\ \text{kg/m}^3)$$
$$V_2 = 25.6\ \text{m}^3 \qquad \textbf{Ans.}$$

Ans:
$\rho_2 = 0.334\ \text{kg/m}^3$
$V_2 = 25.6\ \text{m}^3$

1–22. When a pressure of 650 psi is applied to a solid, its specific weight increases from 310 lb/ft^3 to 312 lb/ft^3. Determine the approximate bulk modulus.

SOLUTION

Differentiating $\mathcal{V} = \dfrac{W}{\gamma}$ with respect to γ, we obtain

$$d\mathcal{V} = -\frac{W}{\gamma^2}\,d\gamma$$

Then

$$E_{\mathcal{V}} = -\frac{dp}{d\mathcal{V}/\mathcal{V}} = -\frac{dp}{\left[-\dfrac{W}{\gamma^2}d\gamma \Big/ (W/\gamma)\right]} = \frac{dp}{d\gamma/\gamma}$$

Therefore,

$$E_{\mathcal{V}} = \frac{650\ \text{lb/in}^2}{\left(\dfrac{(312\ \text{lb/ft}^3 - 310\ \text{lb/ft}^3)}{310\ \text{lb/ft}^3}\right)} = 100.75\left(10^3\right)\ \text{psi} = 101\left(10^3\right)\ \text{psi} \quad \textbf{Ans.}$$

The more precise answer can be obtained from

$$E_{\mathcal{V}} = \frac{\displaystyle\int_{p_i}^{p} dp}{\displaystyle\int_{\gamma_i}^{\gamma}\frac{d\gamma}{\gamma}} = \frac{p - p_i}{\ln\left(\dfrac{\gamma}{\gamma_i}\right)} = \frac{650\ \text{lb/in}^2}{\ln\left(\dfrac{312\ \text{lb/ft}^3}{310\ \text{lb/ft}^3}\right)} = 101.07\left(10^3\right)\ \text{psi} = 101\left(10^3\right)\ \text{psi} \quad \textbf{Ans.}$$

Ans:
$E_{\mathcal{V}} = 101(10^3)$ psi

1–23. Water at 20°C is subjected to a pressure increase of 44 MPa. Determine the percent increase in its density. Take $E_{\mathcal{V}} = 2.20$ GPa.

SOLUTION

$$\frac{\Delta\rho}{\rho_1} = \frac{m/\mathcal{V}_2 - m/\mathcal{V}_1}{m/\mathcal{V}_1} = \frac{\mathcal{V}_1}{\mathcal{V}_2} - 1$$

To find $\mathcal{V}_1/\mathcal{V}_2$, use $E_{\mathcal{V}} = -d_p/(d\mathcal{V}/\mathcal{V})$.

$$\frac{d\mathcal{V}}{\mathcal{V}} = -\frac{dp}{E_{\mathcal{V}}}$$

$$\int_{\mathcal{V}_1}^{\mathcal{V}_2} \frac{d\mathcal{V}}{\mathcal{V}} = -\frac{1}{E_{\mathcal{V}}}\int_{p_1}^{p_2} dp$$

$$\ln\left(\frac{\mathcal{V}_1}{\mathcal{V}_2}\right) = \frac{1}{E_{\mathcal{V}}}\Delta p$$

$$\frac{\mathcal{V}_1}{\mathcal{V}_2} = e^{\Delta p/E_{\mathcal{V}}}$$

So, since the bulk modulus of water at 20°C is $E_{\mathcal{V}} = 2.20$ GPa,

$$\begin{aligned}\frac{\Delta\rho}{\rho_1} &= e^{\Delta p/E_{\mathcal{V}}} - 1\\ &= e^{(44\text{ MPa})/2.20\text{ GPa})} - 1\\ &= 0.0202 = 2.02\%\end{aligned}$$ **Ans.**

Ans:

$$\frac{\Delta\rho}{\rho_1} = 2.02\%$$

***1–24.** If the bulk modulus for water at 70°F is 319 kip/in^2, determine the change in pressure required to reduce its volume by 0.3%.

SOLUTION

Use $E_{V} = -dp/(dV/V)$.

$$dp = -E_{V}\frac{dV}{V}$$

$$\Delta p = \int_{p_i}^{p_f} dp = -E_{V}\int_{V_i}^{V_f}\frac{dV}{V}$$

$$= -(319\ \text{kip/in}^2)\ln\left(\frac{V - 0.03V}{V}\right)$$

$$= 0.958\ \text{kip/in}^2\ (\text{ksi})$$ **Ans.**

Ans:
$\Delta p = 0.958\ \text{kip/in}^2\ (\text{ksi})$

1–25. At a point deep in the ocean, the specific weight of seawater is 64.2 lb/ft^3. Determine the absolute pressure in lb/in^2 at this point if at the surface the specific weight is $\gamma = 63.6\ \text{lb/ft}^3$ and the absolute pressure is $p_a = 14.7\ \text{lb/in}^2$. Take $E_{\forall} = 48.7(10^6)\ \text{lb/ft}^2$.

SOLUTION

Differentiating $\forall = \dfrac{W}{\gamma}$ with respect to γ, we obtain

$$d\forall = -\frac{W}{\gamma^2}\,d\gamma$$

Then

$$E_{\forall} = -\frac{dp}{d\forall/\forall} = -\frac{dp}{\left(-\dfrac{W}{\gamma^2}d\gamma\right)\Big/(W/\gamma)} = \frac{dp}{d\gamma/\gamma}$$

$$dp = E_{\forall}\frac{d\gamma}{\gamma}$$

Integrate this equation with the initial condition at $p = p_a$, $\gamma = \gamma_0$, then

$$\int_{p_a}^{p} dp = E_{\forall}\int_{\gamma_0}^{\gamma}\frac{d\gamma}{\gamma}$$

$$p - p_a = E_{\forall}\ln\frac{\gamma}{\gamma_0}$$

$$p = p_a + E_{\forall}\ln\frac{\gamma}{\gamma_0}$$

Substitute $p_a = 14.7\ \text{lb/in}^2$, $E_{\forall} = \left[48.7(10^6)\,\dfrac{\text{lb}}{\text{ft}^2}\right]\left(\dfrac{1\ \text{ft}}{12\ \text{in}}\right)^2 = 338.19(10^3)\ \text{lb/in}^2$

$\gamma_0 = 63.6\ \text{lb/ft}^3$ and $\gamma = 64.2\ \text{lb/ft}^3$ into this equation,

$$p = 14.7\ \text{lb/in}^2 + [338.19(10^3)\ \text{lb/in}^2]\left[\ln\left(\frac{64.2\ \text{lb/ft}^3}{63.6\ \text{lb/ft}^3}\right)\right]$$

$$= 3.190(10^3)\ \text{psi}$$

$$= 3.19(10^3)\ \text{psi} \qquad \textbf{Ans.}$$

Ans:
$p = 3.19(10^3)$ psi

1–26. A 2-kg mass of oxygen is held at a constant temperature of 50°C and an absolute pressure of 220 kPa. Determine its bulk modulus.

SOLUTION

$$E_{V} = -\frac{dp}{dV/V} = -\frac{dpV}{dV}$$

$$p = \rho RT$$

$$dp = d\rho RT$$

$$E_{V} = -\frac{d\rho RTV}{dV} = -\frac{d\rho pV}{\rho dV}$$

$$\rho = \frac{m}{V}$$

$$d\rho = -\frac{m dV}{V^2}$$

$$E_{V} = \frac{m dV pV}{V^2 (m/V) dV} = P = 220 \text{ kPa} \qquad \textbf{Ans.}$$

Note: This illustrates a general point. For an ideal gas, the isothermal (constant-temperature) bulk modulus equals the absolute pressure.

Ans:
$E_{V} = 220$ kPa

1–27. The viscosity of SAE 10 W30 oil is $\mu = 0.100\ \mathrm{N\cdot s/m^2}$. Determine its kinematic viscosity. The specific gravity is $S_o = 0.92$. Express the answer in SI and FPS units.

SOLUTION

The density of the oil can be determined from

$$\rho_o = S_o\rho_w = 0.92\left(1000\ \mathrm{kg/m^3}\right) = 920\ \mathrm{kg/m^3}$$

Then,

$$\nu_o = \frac{\mu_o}{\rho_o} = \frac{0.100\ \mathrm{N\cdot s/m^2}}{920\ \mathrm{kg/m^3}} = 108.70\left(10^{-6}\right)\mathrm{m^2/s} = 109\left(10^{-6}\right)\mathrm{m^2/s} \qquad \textbf{Ans.}$$

In FPS units,

$$\nu_o = \left[108.70\left(10^{-6}\right)\frac{\mathrm{m^2}}{\mathrm{s}}\right]\left(\frac{1\ \mathrm{ft}}{0.3048\ \mathrm{m}}\right)^2$$

$$= 1.170\left(10^{-3}\right)\mathrm{ft^2/s} = 1.17\left(10^{-3}\right)\mathrm{ft^2/s} \qquad \textbf{Ans.}$$

Ans:
$\nu_o = 109\left(10^{-6}\right)\mathrm{m^2/s}$
$= 1.17\left(10^{-3}\right)\mathrm{ft^2/s}$

***1–28.** If the kinematic viscosity of glycerin is $\nu = 1.15(10^{-3})\ \mathrm{m^2/s}$, determine its viscosity in FPS units. At the temperature considered, glycerin has a specific gravity of $S_g = 1.26$.

SOLUTION

The density of glycerin is

$$\rho_g = S_g \rho_w = 1.26\left(1000\ \mathrm{kg/m^3}\right) = 1260\ \mathrm{kg/m^3}$$

Then,

$$\nu_g = \frac{\mu_y}{\rho_g}; \quad 1.15\left(10^{-3}\right)\ \mathrm{m^2/s} = \frac{\mu_g}{1260\ \mathrm{kg/m^3}}$$

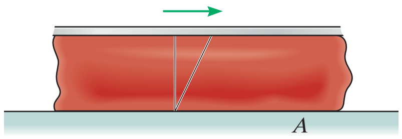
$$\mu_g = \left(1.449\ \frac{\mathrm{N\cdot s}}{\mathrm{m^2}}\right)\left(\frac{1\ \mathrm{lb}}{4.448\ \mathrm{N}}\right)\left(\frac{0.3048\ \mathrm{m}}{1\ \mathrm{ft}}\right)^2$$

$$= 0.03026\ \mathrm{lb\cdot s/ft^2}$$

$$= 0.0303\ \mathrm{lb\cdot s/ft^2} \qquad \textbf{Ans.}$$

Ans:
$\mu_g = 0.0303\ \mathrm{lb\cdot s/ft^2}$

1–29. An experimental test using human blood at $T = 30°\text{C}$ indicates that it exerts a shear stress of $\tau = 0.15\ \text{N/m}^2$ on surface A, where the measured velocity gradient is $16.8\ \text{s}^{-1}$. Since blood is a non-Newtonian fluid, determine its *apparent viscosity* at A.



SOLUTION

Here, $\dfrac{du}{dy} = 16.8\ \text{s}^{-1}$ and $\tau = 0.15\ \text{N/m}^2$. Thus,

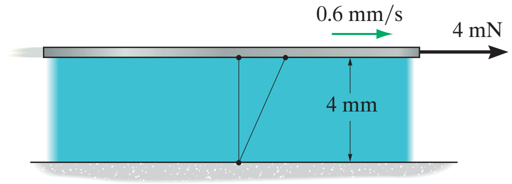
$$\tau = \mu_a \frac{du}{dy}; \qquad 0.15\ \text{N/m}^2 = \mu_a\left(16.8\ \text{s}^{-1}\right)$$

$$\mu_a = 8.93\left(10^{-3}\right)\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Realize that blood is a non-Newtonian fluid. For this reason, we are calculating the *apparent* viscosity.

Ans:
$\mu_a = 8.93(10^{-3})\ \text{N}\cdot\text{s/m}^2$

1–30. The plate is moving at 0.6 mm/s when the force applied to the plate is 4 mN. If the surface area of the plate in contact with the liquid is 0.5 m^2, determine the approximate viscosity of the liquid, assuming that the velocity distribution is linear.



SOLUTION

The shear stress acting on the fluid contact surface is

$$\tau = \frac{F}{A} = \frac{4(10^{-3})\text{ N}}{0.5\text{ m}^2} = 8.00(10^{-3})\text{ N/m}^2$$

Since the velocity distribution is assumed to be linear, the velocity gradient is a constant.

$$\tau = \mu\frac{du}{dy}; \qquad 8.00\,(10^{-3})\text{ N/m}^2 = \mu\left[\frac{0.6(10^{-3})\text{m/s}}{4(10^{-3})\text{m}}\right]$$

$$\mu = 0.0533\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Ans:
$\mu = 0.0533\text{ N}\cdot\text{s/m}^2$

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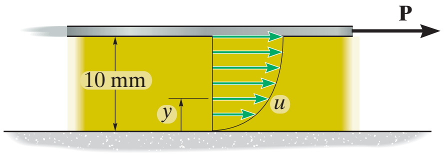
1–31. When the force **P** is applied to the plate, the velocity profile for a Newtonian fluid that is confined under the plate is approximated by $u = (4.23y^{1/3})$ mm/s, where y is in mm. Determine the shear stress within the fluid at $y = 5$ mm. Take $\mu = 0.630(10^{-3})\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = (4.23y^{1/3})\ \mathrm{mm/s}$$

$$\frac{du}{dy} = \left[\frac{1}{3}(4.23)y^{-2/3}\right]\mathrm{s}^{-1}$$

$$= \left(\frac{1.41}{y^{2/3}}\right)\mathrm{s}^{-1}$$

At $y = 5$ mm,

$$\frac{du}{dy} = \left(\frac{1.41}{5^{2/3}}\right)\mathrm{s}^{-1} = 0.4822\ \mathrm{s}^{-1}$$

The shear stress is

$$\tau = \mu\frac{du}{dy} = [0.630(10^{-3})\mathrm{N \cdot s/m^2}](0.4822\ \mathrm{s}^{-1})$$

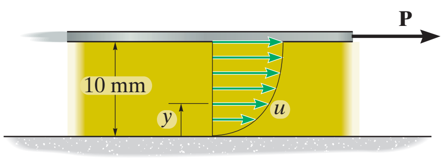
$$= 0.3038(10^{-3})\ \mathrm{N/m^2}$$

$$= 0.304\ \mathrm{mPa}$$ **Ans.**

Note: When $y = 0$, $\frac{du}{dy} \rightarrow \infty$ and so $\tau \rightarrow \infty$. Hence, the equation cannot be applied at this point.

Ans:
$\tau = 0.304$ mPa

***1–32.** When the force **P** is applied to the plate, the velocity profile for a Newtonian fluid that is confined under the plate is approximated by $u = (4.23y^{1/3})$ mm/s, where y is in mm. Determine the minimum shear stress within the fluid. Take $\mu = 0.630(10^{-3})\ \text{N}\cdot\text{s/m}^2$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = (4.23y^{1/3})\ \text{mm/s}$$

$$\frac{du}{dy} = \left[\frac{1}{3}(4.23)y^{-2/3}\right]\text{s}^{-1}$$

$$= \left(\frac{1.41}{y^{2/3}}\right)\text{s}^{-1}$$

The velocity gradient is smallest when $y = 10$ mm and this minimum value is

$$\left(\frac{du}{dy}\right)_{\min} = \left(\frac{1.41}{10^{2/3}}\right)\text{s}^{-1} = 0.3038\ \text{s}^{-1}$$

The minimum shear stress is

$$\tau_{\min} = \mu\left(\frac{du}{dy}\right)_{\min} = \left[0.630\left(10^{-3}\right)\text{N}\cdot\text{s/m}^2\right]\left(0.3038\ \text{s}^{-1}\right)$$

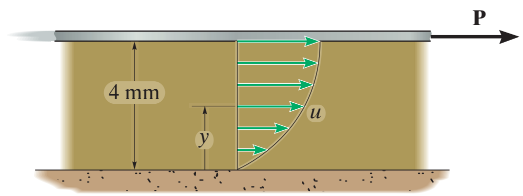
$$= 0.1914\left(10^{-3}\right)\text{N/m}^2$$

$$= 0.191\ \text{MPa}$$ **Ans.**

Note: When $y = 0$, $\frac{du}{dy} \rightarrow \infty$ and so $\tau \rightarrow \infty$. Hence, the equation can not be applied at this point.

Ans:
$\tau_{\min} = 0.191$ MPa

1–33. The Newtonian fluid is confined between a plate and a fixed surface. If its velocity profile is defined by $u = (8y - 0.3y^2)$ mm/s, where y is in mm, determine the shear stress that the fluid exerts on the plate and on the fixed surface. Take $\mu = 0.482\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y.

$$u = \left(8y - 0.3y^2\right)\ \mathrm{mm/s}$$

$$\frac{du}{dy} = (8 - 0.6y)\ \mathrm{s^{-1}}$$

At the plate and the fixed surface, $y = 4$ mm and $y = 0$, respectively. Thus,

$$\left(\frac{du}{dy}\right)_p = [8 - 0.6(4)]\ \mathrm{s^{-1}} = 5.60\ \mathrm{s^{-1}}$$

$$\left(\frac{du}{dy}\right)_{fs} = [8 - 0.6(0)]\ \mathrm{s^{-1}} = 8.00\ \mathrm{s^{-1}}$$

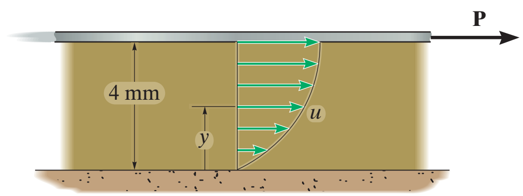
The shear stresses are

$$\tau_p = \mu\left(\frac{du}{dy}\right)_p = \left(0.482\ \mathrm{N \cdot s/m^2}\right)\left(5.60\ \mathrm{s^{-1}}\right) = 2.699\ \mathrm{N/m^2} = 2.70\ \mathrm{Pa}$$

$$\tau_{fs} = \mu\left(\frac{du}{dy}\right)_{fs} = \left(0.482\ \mathrm{N \cdot s/m^2}\right)\left(8.00\ \mathrm{s^{-1}}\right) = 3.856\ \mathrm{N/m^2} = 3.86\ \mathrm{Pa} \quad \textbf{Ans.}$$

Ans:
$\tau_p = 2.70$ Pa
$\tau_{fs} = 3.86$ Pa

1–34. The Newtonian fluid is confined between the plate and a fixed surface. If its velocity profile is defined by $u = (8y - 0.3y^2)$ mm/s, where y is in mm, determine the force **P** that must be applied to the plate to cause this motion. The plate has a surface area of $15(10^3)$ mm^2 in contact with the fluid. Take $\mu = 0.482\ \mathrm{N \cdot s/m^2}$.



SOLUTION

Since the velocity distribution is not linear, the velocity gradient varies with y given by

$$u = \left(8y - 0.3y^2\right)\ \mathrm{mm/s}$$

$$\frac{du}{dy} = (8 - 0.6y)\ \mathrm{s^{-1}}$$

At the plate, $y = 4$ mm. Thus,

$$\left(\frac{du}{dy}\right)_p = [8 - 0.6(4)]\ \mathrm{s^{-1}} = 5.60\ \mathrm{s^{-1}}$$

The shear stress is

$$\tau_p = \mu\left(\frac{du}{dy}\right)_p = \left(0.482\ \mathrm{N \cdot s/m^2}\right)\left(5.60\ \mathrm{s^{-1}}\right) = 2.6992\ \mathrm{N/m^2}$$

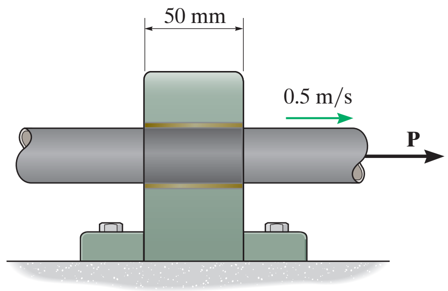
Thus, the force P applied to the plate is

$$P = \tau_p A = (2.6992\ \mathrm{N/m^2})\left[15\left(10^3\right)\ \mathrm{mm^2}\right]\left(\frac{1\ \mathrm{m}}{1000\ \mathrm{mm}}\right)^2$$

$$= 0.04049\ \mathrm{N} = 0.0405\ \mathrm{N} \qquad \textbf{Ans.}$$

Ans:
$P = 0.0405$ N

1–35. If a force of $P = 2$ N causes the 30-mm-diameter shaft to slide along the lubricated bearing with a constant speed of 0.5 m/s, determine the viscosity of the lubricant and the constant speed of the shaft when $P = 8$ N. Assume the lubricant is a Newtonian fluid and the velocity profile between the shaft and the bearing is linear. The gap between the bearing and the shaft is 1 mm.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$\frac{2\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = \mu\left(\frac{0.5\text{ m/s}}{0.001\text{ m}}\right)$$

$$\mu = 0.8498\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

Thus,

$$\frac{8\text{ N}}{[2\pi(0.015\text{ m})](0.05\text{ m})} = (0.8488\text{ N}\cdot\text{s/m}^2)\left(\frac{v}{0.001\text{ m}}\right)$$

$$v = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

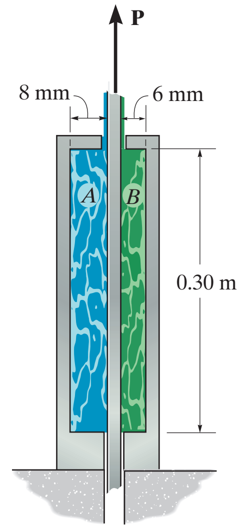
Also, by proportion,

$$\frac{\left(\dfrac{2\text{ N}}{A}\right)}{\left(\dfrac{8\text{ N}}{A}\right)} = \frac{\mu\left(\dfrac{0.5\text{ m/s}}{t}\right)}{\mu\left(\dfrac{v}{t}\right)}$$

$$v = \frac{4}{2}\text{ m/s} = 2.00\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$\mu = 0.849\text{ N}\cdot\text{s/m}^2$
$v = 2.00\text{ m/s}$

***1–36.** A plastic strip having a width of 0.2 m and a mass of 150 g passes between two layers A and B of paint having a viscosity of $5.24\ \mathrm{N\cdot s/m^2}$. Determine the force **P** required to overcome the viscous friction on each side if the strip moves upwards at a constant speed of 4 mm/s. Neglect any friction at the top and bottom openings, and assume the velocity profile through each layer is linear.



SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant. For layers A and B,

$$\left(\frac{du}{dy}\right)_A = \frac{4\ \mathrm{mm/s}}{8\ \mathrm{mm}} = 0.5\ \mathrm{s^{-1}} \quad \left(\frac{du}{dy}\right)_B = \frac{4\ \mathrm{mm/s}}{6\ \mathrm{mm}} = 0.66675\ \mathrm{s^{-1}}$$

The shear stresses acting on the surfaces in contact with layers A and B are

$$\tau_A = \mu\left(\frac{du}{dy}\right)_A = (5.24\ \mathrm{N\cdot s/m^2})(0.5\ \mathrm{s^{-1}}) = 2.62\ \mathrm{N/m^2}$$

$$\tau_B = \mu\left(\frac{du}{dy}\right)_B = (5.24\ \mathrm{N\cdot s/m^2})(0.6667\ \mathrm{s^{-1}}) = 3.4933\ \mathrm{N/m^2}$$

Thus, the shear forces acting on the contact surfaces are

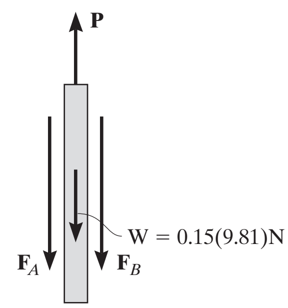
$$F_A = \tau_A A = (2.62\ \mathrm{N/m^2})[(0.2\ \mathrm{m})(0.3\ \mathrm{m})] = 0.1572\ \mathrm{N}$$

$$F_B = \tau_B A = (3.4933\ \mathrm{N/m^2})[(0.2\ \mathrm{m})(0.3\ \mathrm{m})] = 0.2096\ \mathrm{N}$$

Consider the force equilibrium along y axis for the FBD of the strip, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad P - 0.15(9.81)\ \mathrm{N} - 0.1572\ \mathrm{N} - 0.2096\ \mathrm{N} = 0$$

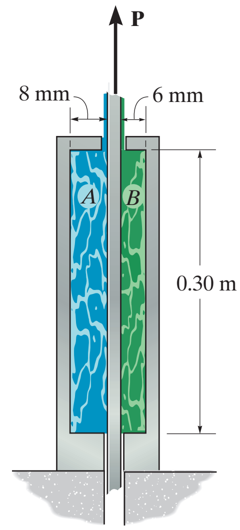
$$P = 1.8383\ \mathrm{N} = 1.84\ \mathrm{N} \qquad \textbf{Ans.}$$



(a)

Ans:
$P = 1.84\ \mathrm{N}$

1–37. A plastic strip having a width of 0.2 m and a mass of 150 g passes between two layers A and B of paint. If force $P = 2\text{ N}$ is applied to the strip, causing it to move at a constant speed of 6 mm/s, determine the viscosity of the paint. Neglect any friction at the top and bottom openings, and assume the velocity profile through each layer is linear.



SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant. For layers A and B,

$$\left(\frac{du}{dy}\right)_A = \frac{6\text{ mm/s}}{8\text{ mm}} = 0.75\text{ s}^{-1} \qquad \left(\frac{du}{dy}\right)_B = \frac{6\text{ mm/s}}{6\text{ mm}} = 1.00\text{ s}^{-1}$$

The shear stresses acting on the surfaces of the strip in contact with layers A and B are

$$\tau_A = \mu\left(\frac{du}{dy}\right)_A = \mu(0.75\text{ s}^{-1}) = (0.75\mu)\text{ N/m}^2$$

$$\tau_B = \mu\left(\frac{du}{dy}\right)_B = \mu(1.00\text{ s}^{-1}) = (1.00\mu)\text{ N/m}^2$$

Thus, the shear forces acting on these contact surfaces are

$$F_A = \tau_A A = (0.75\mu\text{ N/m}^2)[(0.2\text{ m})(0.3\text{ m})] = (0.045\mu)\text{ N}$$

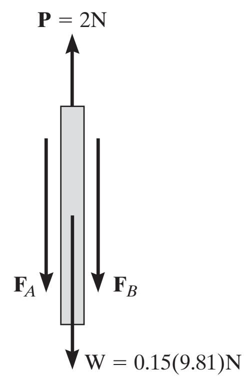
$$F_B = \tau_B A = (1.00\mu\text{ N/m}^2)[(0.2\text{ m})(0.3\text{ m})] = (0.06\mu)\text{ N}$$

Consider the force equilibrium along the y axis for the FBD of the strip, Fig. a.

$$+\uparrow \Sigma F_y = 0; \qquad 2\text{ N} - 0.045\mu\text{ N} - 0.06\mu\text{ N} - 0.15(9.81)\text{ N} = 0$$

$$\mu = 5.0333\text{ N}\cdot\text{s/m}^2$$

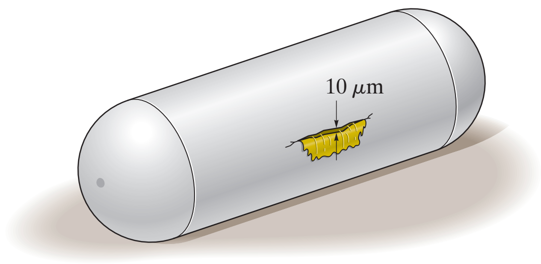
$$= 5.03\text{ N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$



(a)

Ans:
$\mu = 5.03\text{ N}\cdot\text{s/m}^2$

1–38. The tank containing gasoline has a long crack on its side that has an average opening of 10 μm. The velocity through the crack is approximated by the equation $u = 10(10^9)\left[10(10^{-6})y - y^2\right]$ m/s, where y is in meters, measured upward from the bottom of the crack. Find the shear stress at the bottom, $y = 0$, and the location y within the crack where the shear stress in the gasoline is zero. Take $\mu_g = 0.317(10^{-3})\ \mathrm{N\cdot s/m^2}$.



SOLUTION

Gasoline is a Newtonian fluid.

The rate of change of shear strain as a function of y is

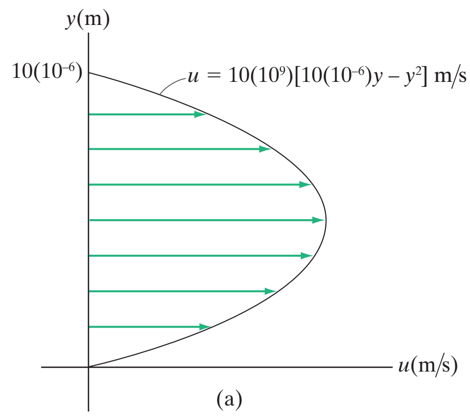
$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right]\ \mathrm{s^{-1}}$$

At the surface of crack, $y = 0$ and $y = 10(10^{-6})$ m. Then

$$\left.\frac{du}{dy}\right|_{y=0} = 10(10^9)\left[10(10^{-6}) - 2(0)\right] = 100(10^3)\ \mathrm{s^{-1}}$$

or

$$\left.\frac{du}{dy}\right|_{y=10(10^{-6})\,m} = 10(10^9)\left\{10(10^{-6}) - 2\left[10(10^{-6})\right]\right\} = -100(10^3)\ \mathrm{s^{-1}}$$



(a)

Applying Newton's law of viscosity,

$$\tau_{y=0} = \mu_g \left.\frac{du}{dy}\right|_{y=0} = \left[0.317(10^{-3})\ \mathrm{N\cdot s/m^2}\right]\left[100(10^3)\ \mathrm{s^{-1}}\right] = 31.7\ \mathrm{N/m^2} \qquad \textbf{Ans.}$$

$\tau = 0$ when $\dfrac{du}{dy} = 0$. Thus,

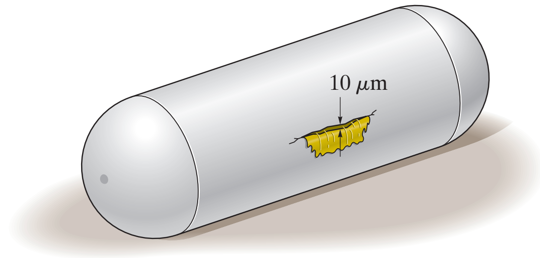
$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right] = 0$$

$$10(10^{-6}) - 2y = 0$$

$$y = 5(10^{-6})\ \mathrm{m} = 5\ \mu\mathrm{m} \qquad \textbf{Ans.}$$

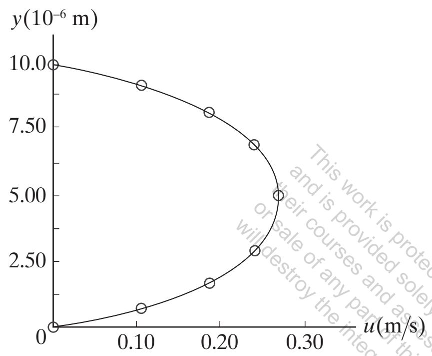
Ans:
$\tau_{y=0} = 31.7\ \mathrm{N/m^2}$
$\tau = 0$ when $y = 5\ \mu\mathrm{m}$

1–39. The tank containing gasoline has a long crack on its side that has an average opening of 10 μm. If the velocity profile through the crack is approximated by the equation $u = 10(10^9)[10(10^{-6})y - y^2]$ m/s, where y is in meters, plot both the velocity profile and the shear stress distribution for the gasoline as it flows through the crack. Take $\mu_g = 0.317(10^{-3})$ N·s/m².



SOLUTION

$y(10^{-6}\text{ m})$	0	1.25	2.50	3.75	5.00
$u(\text{m/s})$	0	0.1094	0.1875	0.2344	0.250
	6.25	7.50	8.75	10.0	
	0.2344	0.1875	0.1094	0	



(a)

Gasoline is a Newtonian fluid. The rate of change of shear strain as a function of y is

$$\frac{du}{dy} = 10(10^9)\left[10(10^{-6}) - 2y\right] \text{ s}^{-1}$$

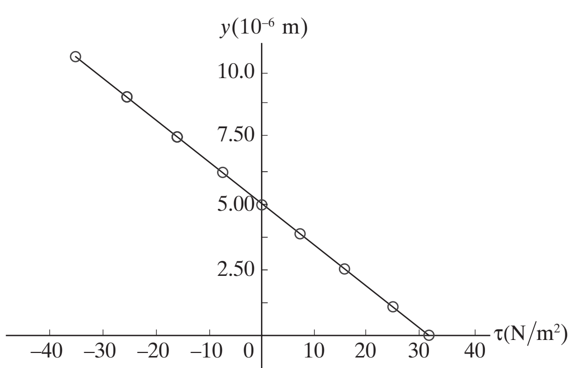
Applying Newton's law of viscoscity,

$$\tau = \mu\frac{du}{dy} = \left[0.317(10^{-3}) \text{ N}\cdot\text{s/m}^2\right]\left\{10(10^9)\left[10(10^{-6}) - 2y\right] \text{ s}^{-1}\right\}$$

$$\tau = 3.17(10^6)\left[10(10^{-6}) - 2y\right] \text{ N/m}^2$$

The plots of the velocity profile and the shear stress distribution are shown in Figs. *a* and *b*, respectively.

$y(10^{-6}\text{ m})$	0	1.25	2.50	3.75	5.00
$\tau(\text{N/m}^2)$	31.70	23.78	15.85	7.925	0
	6.25	7.50	8.75	10.0	
	−7.925	−15.85	−23.78	−31.70	



(b)

***1–40.** Determine the constants B and C in Andrade's equation for water if it has been experimentally determined that $\mu = 1.00(10^{-3})\ \mathrm{N \cdot s/m^2}$ at a temperature of 20°C and that $\mu = 0.554(10^{-3})\ \mathrm{N \cdot s/m^2}$ at 50°C.

SOLUTION

The Andrade's equation is

$$\mu = Be^{C/T}$$

At $T = (20 + 273)\ \mathrm{K} = 293\ \mathrm{K}$, $\mu = 1.00\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2}$. Thus,

$$\begin{aligned}
1.00\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2} &= Be^{C/293\ \mathrm{K}} \\
\ln\left[1.00\left(10^{-3}\right)\right] &= \ln\left(Be^{C/293}\right) \\
-6.9078 &= \ln B + \ln e^{C/293} \\
-6.9078 &= \ln B + C/293 \\
\ln B &= -6.9078 - C/293 \qquad \textbf{(1)}
\end{aligned}$$

At $T = (50 + 273)\ \mathrm{K} = 323\ \mathrm{K}$, $\mu = 0.554\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2}$. Thus,

$$\begin{aligned}
0.554\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2} &= Be^{C/323} \\
\ln\left[0.554\left(10^{-3}\right)\right] &= \ln\left(Be^{C/323}\right) \\
-7.4983 &= \ln B + \ln e^{C/323} \\
-7.4983 &= \ln B + \frac{C}{323} \\
\ln B &= -7.4983 - \frac{C}{323} \qquad \textbf{(2)}
\end{aligned}$$

Equating Eqs. (1) and (2),

$$\begin{aligned}
-6.9078 - \frac{C}{293} &= -7.4983 - \frac{C}{323} \\
0.5906 &= 0.31699\left(10^{-3}\right) C \\
C &= 1863.10 = 1863\ \mathrm{K} \qquad \textbf{Ans.}
\end{aligned}$$

Substitute this result into Eq. (1).

$$\begin{aligned}
B &= 1.7316\left(10^{-6}\right)\ \mathrm{N \cdot s/m^2} \\
&= 1.73\left(10^{-6}\right)\ \mathrm{N \cdot s/m^2} \qquad \textbf{Ans.}
\end{aligned}$$

Ans:
$C = 1863\ \mathrm{K}$
$B = 1.73\left(10^{-6}\right)\ \mathrm{N \cdot s/m^2}$

1–41. The viscosity of water can be determined using the empirical Andrade's equation with the constants $B = 1.732(10^{-6})\ \mathrm{N \cdot s/m^2}$ and $C = 1863\ \mathrm{K}$. With these constants, compare the results of using this equation with those tabulated in Appendix A for temperatures of $T = 10°\mathrm{C}$ and $T = 80°\mathrm{C}$.

SOLUTION

The Andrade's equation for water is

$$\mu = 1.732\left(10^{-6}\right)e^{1863/T}$$

At $T = (10 + 273)\ \mathrm{K} = 283\ \mathrm{K}$,

$$\mu = 1.732\left(10^{-6}\right)e^{1863\ \mathrm{K}/283\ \mathrm{K}} = 1.25\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2} \qquad \textbf{Ans.}$$

From the Appendix at $T = 10°\mathrm{C}$,

$$\mu = 1.31\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2}$$

At $T = (80 + 273)\ \mathrm{K} = 353\ \mathrm{K}$,

$$\mu = 1.732\left(10^{-6}\right)e^{1863\ \mathrm{K}/353\ \mathrm{K}} = 0.339\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2} \qquad \textbf{Ans.}$$

From the Appendix at $T = 80°\mathrm{C}$,

$$\mu = 0.356\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2}$$

Ans:
At $T = 283\ \mathrm{K}$, $\mu = 1.25\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2}$
At $T = 353\ \mathrm{K}$, $\mu = 0.339\left(10^{-3}\right)\ \mathrm{N \cdot s/m^2}$

1–42. Determine the constants B and C in the Sutherland equation for air if it has been experimentally determined that at standard atmospheric pressure and a temperature of 20°C, $\mu = 18.3(10^{-6})\ \mathrm{N \cdot s/m^2}$, and at 50°C, $\mu = 19.6(10^{-6})\ \mathrm{N \cdot s/m^2}$.

SOLUTION

The Sutherland equation is

$$\mu = \frac{BT^{3/2}}{T + C}$$

At $T = (20 + 273)\ \mathrm{K} = 293\ \mathrm{K}$, $\mu = 18.3(10^{-6})\ \mathrm{N \cdot s/m^2}$. Thus,

$$18.3(10^{-6})\ \mathrm{N \cdot s/m^2} = \frac{B(293^{3/2})}{293\ \mathrm{K} + C}$$

$$B = 3.6489(10^{-9})(293 + C) \qquad \textbf{(1)}$$

At $T = (50 + 273)\ \mathrm{K} = 323\ \mathrm{K}$, $\mu = 19.6(10^{-6})\ \mathrm{N \cdot s/m^2}$. Thus,

$$19.6(10^{-6})\ \mathrm{N \cdot s/m^2} = \frac{B(323^{3/2})}{323\ \mathrm{K} + C}$$

$$B = 3.3764(10^{-9})(323 + C) \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$B = 1.36(10^{-6})\ \mathrm{N \cdot s/(m^2 \cdot K^{\frac{1}{2}})}$ $\qquad C = 78.8\ \mathrm{K}$ **Ans.**

Ans:
$B = 1.36(10^{-6})\ \mathrm{N \cdot s/(m^2 \cdot K^{\frac{1}{2}})}$, $C = 78.8\ \mathrm{K}$

1–43. The constants $B = 1.357(10^{-6})\ \text{N}\cdot\text{s}/(\text{m}^2\cdot\text{K}^{1/2})$ and $C = 78.84$ K have been used in the empirical Sutherland equation to determine the viscosity of air at standard atmospheric pressure. With these constants, compare the results of using this equation with those tabulated in Appendix A for temperatures of $T = 10°\text{C}$ and $T = 80°\text{C}$.

SOLUTION

The Sutherland Equation for air at standard atmospheric pressure is

$$\mu = \frac{1.357\left(10^{-6}\right)T^{3/2}}{T + 78.84}$$

At $T = (10 + 273)\ \text{K} = 283\ \text{K}$,

$$\mu = \frac{1.357\left(10^{-6}\right)\left(283^{3/2}\right)}{283 + 78.84} = 17.9\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

From Appendix A at $T = 10°\text{C}$,

$$\mu = 17.6\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2$$

At $T = (80 + 273)\ \text{K} = 353\ \text{K}$,

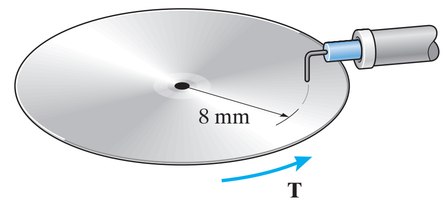
$$\mu = \frac{1.357\left(10^{-6}\right)\left(353^{3/2}\right)}{353 + 78.84} = 20.8\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2 \qquad \textbf{Ans.}$$

From Appendix A at $T = 80°\text{C}$,

$$\mu = 20.9\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2$$

Ans:
Using the Sutherland equation,
at $T = 283\ \text{K}$, $\mu = 17.9\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2$
at $T = 353\ \text{K}$, $\mu = 20.8\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2$

***1–44.** The read–write head for a hand-held music player has a surface area of 0.04 mm^2. The head is held 0.04 μm above the disk, which is rotating at a constant rate of 1800 rpm. Determine the torque **T** that must be applied to the disk to overcome the frictional shear resistance of the air between the head and the disk. The surrounding air is at standard atmospheric pressure and a temperature of 20°C. Assume the velocity profile is linear.



SOLUTION

Here air is a Newtonian fluid.

$$\omega = \left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 60\pi\ \text{rad/s}.$$

Thus, the velocity of the air on the disk is $U = \omega r = (60\pi)(0.008) = 0.48\pi$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. *a*,

$$\frac{du}{dy} = \frac{U}{t} = \frac{0.48\pi\ \text{m/s}}{0.04\left(10^{-6}\right)\text{m}} = 12\left(10^{6}\right)\pi\ \text{s}^{-1}$$

For air at $T = 20°\text{C}$ and standard atmospheric pressure, $\mu = 18.1\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2$ (Appendix A). Applying Newton's law of viscosity,

$$\tau = \mu\frac{du}{dy} = \left[18.1\left(10^{-6}\right)\ \text{N}\cdot\text{s/m}^2\right]\left[12\left(10^{6}\right)\pi\ \text{s}^{-1}\right] = 217.2\pi\ \text{N/m}^2$$

Then, the drag force produced is

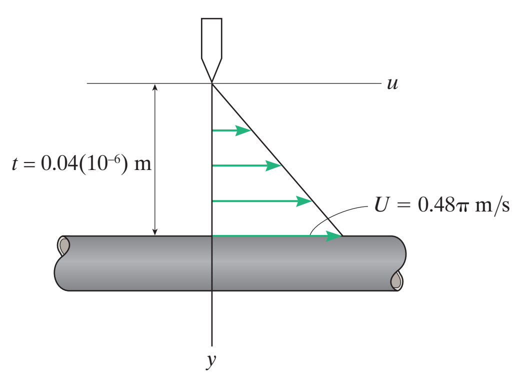
$$F_D = \tau A = \left(217.2\pi\ \text{N/m}^2\right)\left(\frac{0.04}{1000^2}\ \text{m}^2\right) = 8.688\left(10^{-6}\right)\pi\ \text{N}$$

The moment equilibrium about point O requires

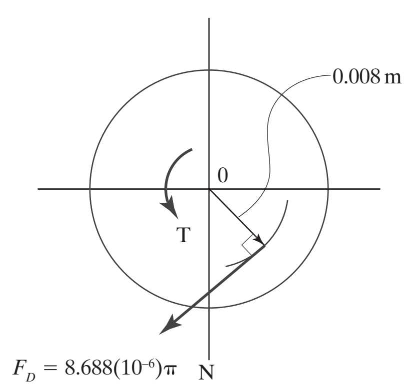
$$\circlearrowleft + \Sigma M_O = 0;\quad T - \left[8.688\left(10^{-6}\right)\pi\ \text{N}\right](0.008\ \text{m}) = 0$$

$$T = 0.218\left(10^{-6}\right)\ \text{N}\cdot\text{m}$$

$$= 0.218\ \mu\text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



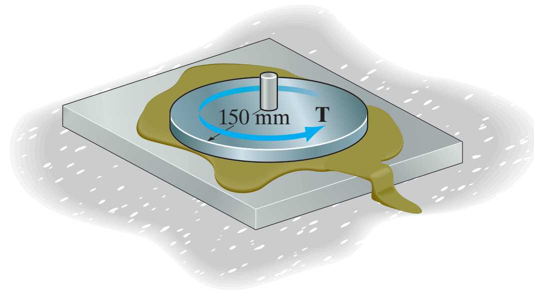
(a)



(b)

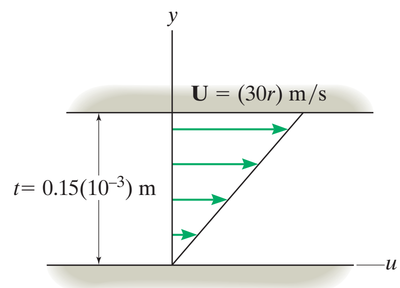
Ans:
$T = 0.218\ \mu\text{N}\cdot\text{m}$

1–45. Determine the torque **T** required to rotate the disk with a constant angular velocity of $\omega = 30$ rad/s. The oil has a thickness of 0.15 mm. Assume the velocity profile is linear, and $\mu = 0.428\ \text{N}\cdot\text{s/m}^2$.



SOLUTION

Oil is a Newtonian fluid. The speed of the oil on the bottom contact surface of the disk is $U = wr = (30r)$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. a, the velocity gradient will be constant given by



(a)

$$\frac{du}{dy} = \frac{V}{t} = \frac{(30r)\ \text{m/s}}{0.15(10^{-3})\ \text{m}} = [200(10^3)r]\ \text{s}^{-1}$$

So then,

$$\tau = \mu\frac{du}{dy} = (0.428\ \text{N}\cdot\text{s/m}^2)[200(10^3)r]\ \text{s}^{-1}$$

$$= [85.6(10^3)r]\ \text{N/m}^2$$

The shaded differential element shown in Fig. b has an area of $dA = 2\pi r dr$. Thus, $dF = \tau dA = (85.6(10^3)r)(2\pi r dr) = 171.2\pi(10^3)r^2 dr$. Moment equilibrium about point O in Fig. b requires

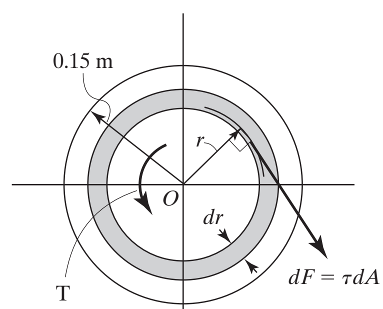
$$\circlearrowleft + \Sigma M_o = 0; \qquad T - \int r dF = 0$$

$$T - \int_0^{0.15m} r[171.2\pi(10^3)r^2 dr] = 0$$

$$T = \int_0^{0.15m} 171.2\pi(10^3)r^3 dr$$

$$= 171.2\pi(10^3)\left(\frac{r^4}{4}\right)\Bigg|_0^{0.15m}$$

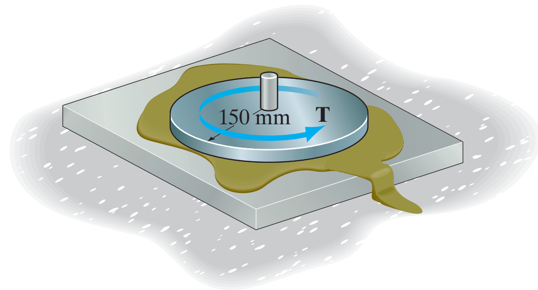
$$= 68.07\ \text{N}\cdot\text{m} = 68.1\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$



(b)

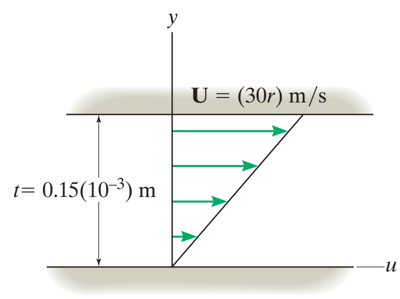
Ans:
$T = 68.1\ \text{N}\cdot\text{m}$

1–46. Determine the torque **T** required to rotate the disk with a constant angular velocity of $\omega = 30$ rad/s as a function of the oil thickness t. Plot your results of torque (vertical axis) versus the oil thickness for $0 \le t \le 0.15(10^{-3})$ m for values every 0.03 (10^{-3}) m. Assume the velocity profile is linear, and $\mu = 0.428\ \mathrm{N \cdot s/m^2}$.



SOLUTION

$t(10^{-3})$ m	0	0.03	0.06	0.09	0.12	0.15
$T(\mathrm{N \cdot m})$	∞	340	170	113	85.1	68.1



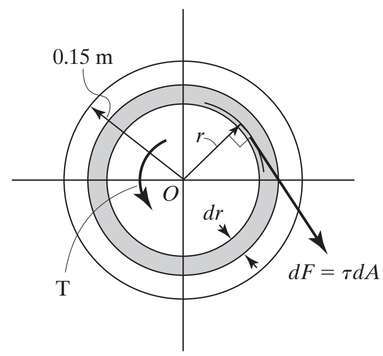
(a)

Oil is a Newtonian fluid. The speed of the oil on the bottom contact surface of the disk is $U = wr = (30r)$ m/s. Since the velocity profile is assumed to be linear as shown in Fig. a, the velocity gradient will be constant, given by

$$\frac{du}{dy} = \frac{U}{t} = \frac{(30r)\ \mathrm{m/s}}{t} = \left(\frac{30r}{t}\right)\mathrm{s^{-1}}$$

So then,

$$\tau = \mu\frac{du}{dy} = (0.428\ \mathrm{N \cdot s/m^2})\left(\frac{30r}{t}\ \mathrm{s^{-1}}\right) = \left(\frac{12.84r}{t}\right)\mathrm{N/m^2}$$



(b)

The shaded differential element shown in Fig. b has an area of $dA = 2\pi r dr$. Thus, $dF = \tau dA = \left(\frac{12.84r}{t}\right)(2\pi r dr) = \frac{25.68\pi}{t}r^2 dr$. Moment equilibrium about point O in Fig. b requires

$$\circlearrowleft + \Sigma M_o = 0; \quad T - \int r dF = 0$$

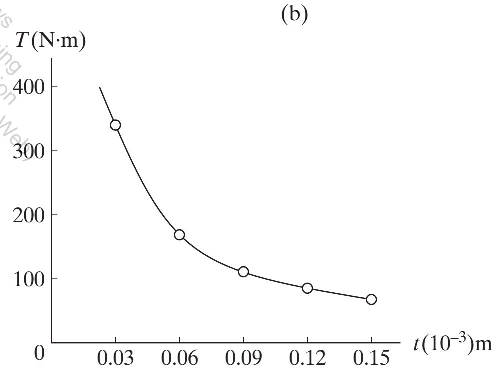
$$T - \int_0^{0.15m} r\left(\frac{25.68\pi}{t}r^2 dr\right) = 0$$

$$T = \int_0^{0.15m} \frac{25.68\pi}{t}r^3 dr = 0$$

$$= \frac{25.68\pi}{t}\left(\frac{r^4}{4}\right)\Bigg|_0^{0.15m}$$

$$= \left(\frac{0.010211}{t}\right)\mathrm{N \cdot m} = \left(\frac{0.0102}{t}\right)\mathrm{N \cdot m} \quad \text{where } t \text{ is in meters.}$$

Ans.



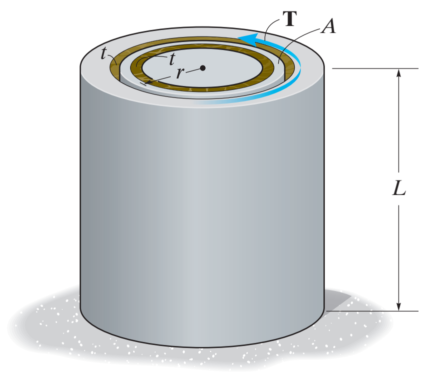
(c)

The plot of T vs t is shown in Fig. c.

Ans:

$T = \left(\frac{0.0102}{t}\right)\mathrm{N \cdot m}$ where t is in meters.

1–47. The very thin tube A of mean radius r and length L is placed within the fixed circular cavity as shown. If the cavity has a small gap of thickness t on each side of the tube, and is filled with a Newtonian liquid having a viscosity μ, determine the torque **T** required to overcome the fluid resistance and rotate the tube with a constant angular velocity of ω. Assume the velocity profile within the liquid is linear.

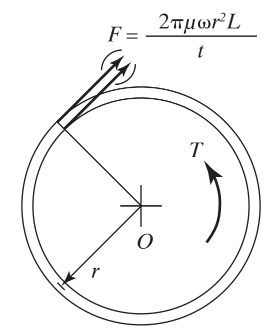


SOLUTION

Since the velocity distribution is assumed to be linear, the velocity gradient will be constant.

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{(\omega r)}{t}$$



(a)

Considering the moment equilibrium of the tube, Fig. a,

$$\Sigma M = 0; \qquad T - 2\tau A r = 0$$

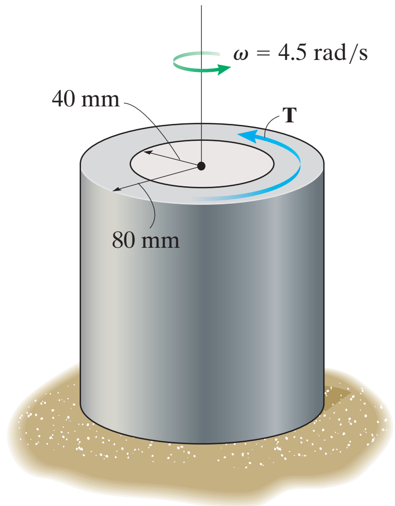
$$T = 2(\mu)\frac{(\omega r)}{t}(2\pi r L)r$$

$$T = \frac{4\pi\mu\omega r^3 L}{t} \qquad \textbf{Ans.}$$

Ans:

$T = \frac{4\pi\mu\omega r^3 L}{t}$

***1–48.** The tube rests on a 1.5-mm thin film of oil having a viscosity of $\mu = 0.0586\ \text{N}\cdot\text{s/m}^2$. If the tube is rotating at a constant angular velocity of $\omega = 4.5\ \text{rad/s}$, determine the shear stress in the oil at $r = 40$ mm and $r = 80$ mm. Assume the velocity profile within the oil is linear.



SOLUTION

Oil is a Newtonian fluid. Since the velocity profile is assumed to be linear, the velocity gradient will be constant. At $r = 40$ mm and $r = 80$ mm,

$$\left.\frac{du}{dy}\right|_{r=40\text{ mm}} = \frac{wr}{t} = \frac{(4.5\ \text{rad/s})(40\ \text{mm})}{1.5\ \text{mm}}$$
$$= 120\ \text{s}^{-1}$$

$$\left.\frac{du}{dy}\right|_{r=80\text{ mm}} = \frac{wr}{t} = \frac{(4.5\ \text{rad/s})(80\ \text{mm})}{1.5\ \text{mm}}$$
$$= 240\ \text{s}^{-1}$$

Then the shear stresses in the oil at $r = 40$ mm and 80 mm are

$$\tau\big|_{r=40\text{ mm}} = \mu\left.\left(\frac{du}{dy}\right)\right|_{r=40\text{ mm}} = \left(0.0586\ \text{N}\cdot\text{s/m}^2\right)\left(120\ \text{s}^{-1}\right) = 7.032\ \text{N/m}^2 = 7.03\ \text{Pa}$$

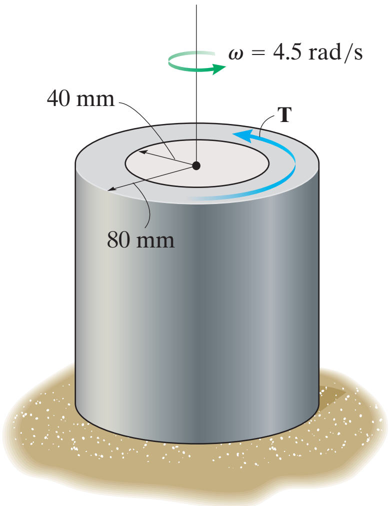
Ans.

$$\tau\big|_{r=80\text{ mm}} = \mu\left.\left(\frac{du}{dy}\right)\right|_{r=80\text{ mm}} = \left(0.0586\ \text{N}\cdot\text{s/m}^2\right)\left(240\ \text{s}^{-1}\right) = 14.064\ \text{N/m}^2 = 14.1\ \text{Pa}$$

Ans.

Ans:
$\tau\big|_{r=40\text{ mm}} = 7.03\ \text{Pa}$
$\tau\big|_{r=80\text{ mm}} = 14.1\ \text{Pa}$

1–49. The tube rests on a 1.5-mm thin film of oil having a viscosity of $\mu = 0.0586\ \mathrm{N \cdot s/m^2}$. If the tube is rotating at a constant angular velocity of $\omega = 4.5\ \mathrm{rad/s}$, determine the torque **T** that must be applied to the tube to maintain the motion. Assume the velocity profile within the oil is linear.



SOLUTION

Oil is a Newtonian fluid. Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus, $\frac{du}{dy} = \frac{U}{t} = \frac{wr}{t}$.

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

Thus, the shear force the oil exerts on the differential element of area $dA = 2\pi r\, dr$ shown shaded in Fig. *a* is

$$dF = \tau dA = \left(\frac{\mu \omega r}{t}\right)(2\pi r\, dr) = \frac{2\pi \mu \omega}{t} r^2 dr$$

Considering the moment equilibrium of the tube about point D,

$$\circlearrowleft + \Sigma M_O = 0; \qquad \int_{r_i}^{r_o} r dF - T = 0$$

$$T = \int_{r_i}^{r_o} r dF = \frac{2\pi \mu \omega}{t} \int_{r_i}^{r_o} r^3 dr$$

$$= \frac{2\pi \mu \omega}{t} \left(\frac{r^4}{4}\right)\Bigg|_{r_i}^{r_o} = \frac{\pi \mu \omega}{2t}\left(r_o^4 - r_i^4\right)$$

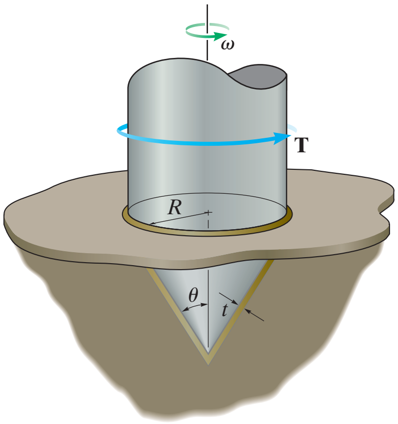
Substituting the numerical values,

$$T = \frac{\pi(0.0586\ \mathrm{N \cdot s/m^2})(4.5\ \mathrm{rad/s})}{2[1.5\ (10^{-3})\ \mathrm{m}]}\left(0.08^4 - 0.04^4\right)$$

$$= 0.01060\ \mathrm{N \cdot m} = 0.0106\ \mathrm{N \cdot m} \qquad \textbf{Ans.}$$

Ans:
$T = 0.0106\ \mathrm{N \cdot m}$

1–50. The conical bearing is placed in a lubricating Newtonian fluid having a viscosity μ. Determine the torque **T** required to rotate the bearing with a constant angular velocity of ω. Assume the velocity profile along the thickness t of the fluid is linear.



SOLUTION

Since the velocity distribution is linear, the velocity gradient will be constant. The velocity of the oil in contact with the shaft at an arbitrary point is $U = \omega r$. Thus,

$$\tau = \mu \frac{du}{dy} = \frac{\mu \omega r}{t}$$

From the geometry shown in Fig. a,

$$z = \frac{r}{\tan\theta} \qquad dz = \frac{dr}{\tan\theta} \qquad \textbf{(1)}$$

Also, from the geometry shown in Fig. b,

$$dz = ds\cos\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2),

$$\frac{dr}{\tan\theta} = ds\cos\theta \qquad ds = \frac{dr}{\sin\theta}$$

The area of the surface of the differential element shown shaded in Fig. a is $dA = 2\pi r ds = \dfrac{2\pi}{\sin\theta} r dr$. Thus, the shear force the oil exerts on this area is

$$dF = \tau dA = \left(\frac{\mu\omega r}{t}\right)\left(\frac{2\pi}{\sin\theta} r dr\right) = \frac{2\pi\mu\omega}{t\sin\theta} r^2 dr$$

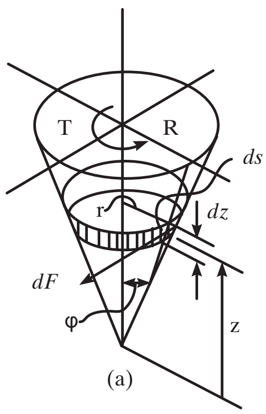
Considering the moment equilibrium of the shaft, Fig. a,

$$\Sigma M_z = 0; \qquad T - \int r dF = 0$$

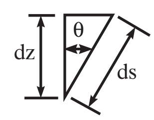
$$T = \int r dF = \frac{2\pi\mu\omega}{t\sin\theta}\int_0^R r^3 dr$$

$$= \frac{2\pi\mu\omega}{t\sin\theta}\left(\frac{r^4}{4}\right)\Bigg|_0^R$$

$$= \frac{\pi\mu\omega R^4}{2t\sin\theta} \qquad \textbf{Ans.}$$



(a)



(b)

Ans:

$$T = \frac{\pi\mu\omega R^4}{2t\sin\theta}$$

1–51. The city of Denver, Colorado is at an elevation of 1610 m above sea level. Determine how hot one can prepare boiling water to make a cup of tea.

SOLUTION

At the elevation of 1610 meters, the atmospheric pressure can be obtained by interpolating the data given in Appendix A.

$$p_{\text{atm}} = 89.88\text{ kPa} - \left(\frac{89.88\text{ kPa} - 79.50\text{ kPa}}{1000\text{ m}}\right)(610\text{ m}) = 83.55\text{ kPa}$$

Since water boils if the vapor pressure is equal to the atmospheric pressure, then the boiling temperature at Denver can be obtained by interpolating the data given in Appendix A.

$$T_{\text{boil}} = 90°\text{C} + \left(\frac{83.55 - 70.1}{84.6 - 70.1}\right)(5°\text{C}) = 94.6°\text{C} \qquad \textbf{Ans.}$$

Note: Compare this with $T_{\text{boil}} = 100°\text{C}$ at 1 atm.

Ans:
$T_{\text{boil}} = 94.6°\text{C}$

***1–52.** How hot can you make a cup of tea if you climb to the top of Mt. Everest (29,000 ft) and attempt to boil water?

SOLUTION

At the elevation of 29 000 ft, the atmospheric pressure can be obtained by interpolating the data given in Appendix A:

$$p_{\text{atm}} = 704.4\ \text{lb/ft}^2 - \left(\frac{704.4\ \text{lb/ft}^2 - 629.6\ \text{lb/ft}^2}{30\,000\ \text{lb/ft}^2 - 27\,500\ \text{lb/ft}^2}\right)(29\,000\ \text{ft} - 27\,500\ \text{ft})$$

$$= \left(659.52\ \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2 = 4.58\ \text{psi} \qquad \textbf{Ans.}$$

Since water boils if the vapor pressure equals the atmospheric pressure, the boiling temperature of the water at Mt. Everest can be obtained by interpolating the data of Appendix A.

$$T_{\text{boil}} = 150°\text{F} + \left(\frac{4.58\ \text{psi} - 3.72\ \text{psi}}{4.75\ \text{psi} - 3.72\ \text{psi}}\right)(160 - 150)°\text{F} = 158°\text{F} \qquad \textbf{Ans.}$$

Note: Compare this with 212°F at 1 atm.

Ans:
$p_{\text{atm}} = 4.58$ psi, $T_{\text{boil}} = 158°\text{F}$

1–53. A boat propeller is rotating in water that has a temperature of 15°C. What is the lowest absolute water pressure that can be developed at the blades so that cavitation will not occur?

SOLUTION

From the table in Appendix A, the vapor pressure of water at $T = 15°\text{C}$ is

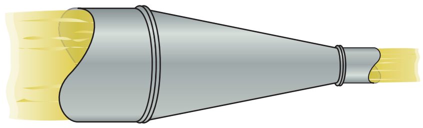
$$p_v = 1.71 \text{ kPa}$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v. Thus,

$$p_{\min} = p_v = 1.71 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$p_{\min} = 1.71 \text{ kPa}$

1–54. As water at 20°C flows through the transition, its pressure will begin to decrease. Determine the lowest absolute pressure it can have without causing cavitation.



SOLUTION

From the table in Appendix A, the vapor pressure of water at $T = 20°\text{C}$ is

$$p_v = 2.34 \text{ kPa}$$

Cavitation (or boiling of water) will occur when the water pressure is equal to or less than p_v. Thus,

$$p_{\text{min}} = 2.34 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$p_{\text{min}} = 2.34 \text{ kPa}$

1–55. Water at 70°F is flowing through a garden hose. If the hose is bent, a hissing noise can be heard. Here cavitation has occurred in the hose because the velocity of the flow has increased at the bend, and the pressure has dropped. What would be the highest absolute pressure in the hose at this location?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 70°\text{F}$ is

$$p_v = 0.363\ \text{lb/in}^2$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v.

$$p_{\max} = p_v = 0.363\ \text{lb/in}^2 \qquad \textbf{Ans.}$$

Ans:
$p_{\max} = 0.363\ \text{lb/in}^2$

***1–56.** Water at 25°C is flowing through a garden hose. If the hose is bent, a hissing noise can be heard. Here cavitation has occurred in the hose because the velocity of the flow has increased at the bend, and the pressure has dropped. What would be the highest absolute pressure in the hose at this location?

SOLUTION

From Appendix A, the vapor pressure of water at $T = 25°\text{C}$ is

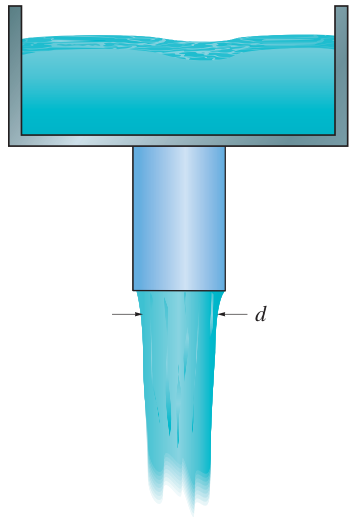
$$p_v = 3.17 \text{ kPa}$$

Cavitation (boiling of water) will occur if the water pressure is equal or less than p_v.

$$p_{\max} = p_v = 3.17 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$p_{\max} = 3.17 \text{ kPa}$

1–57. For water falling out of the tube, there is a difference in pressure Δp between a point located just inside and a point just outside of the stream due to the effect of surface tension σ. Determine the diameter d of the stream at this location.



SOLUTION

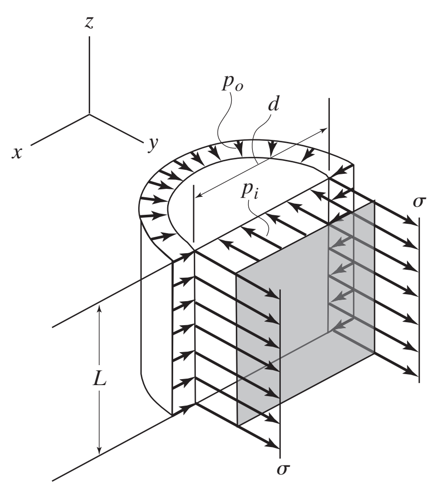
Consider a length L of the water column. The free-body diagram of half of this column is shown in Fig. *a*. Consider the force equilibrium along the y-axis,

$$\Sigma F_y = 0; \quad 2\sigma L + p_o[d(L)] - p_i[d(L)] = 0$$

$$2\sigma = (p_i - p_o)d$$

However, $p_i - p_o = \Delta p$. Then

$$d = \frac{2\sigma}{\Delta p} \qquad \textbf{Ans.}$$



(a)

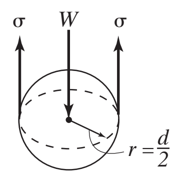
Ans:
$d = \dfrac{2\sigma}{\Delta p}$

1–58. Steel particles are ejected from a grinder and fall gently into a tank of water. Determine the largest average diameter of a particle that will float on the water with a contact angle of $\theta = 180°$. Take $\gamma_{st} = 490\ \text{lb/ft}^3$ and $\sigma = 0.00492\ \text{lb/ft}$. Assume that each particle has the shape of a sphere, where $V = \frac{4}{3}\pi r^3$.

SOLUTION

The weight of a steel particle is

$$W = \gamma_{st}V = \left(490\ \text{lb/ft}^3\right)\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right] = \frac{245\pi}{3}d^3$$



(a)

Force equilibrium along the vertical, Fig. *a*, requires

$$+\uparrow \Sigma F_y = 0; \qquad (0.00492\ \text{lb/ft})\left[2\pi\left(\frac{d}{2}\right)\right] - \frac{245\pi}{3}d^3 = 0$$

$$0.00492\pi d = \frac{245\pi}{3}d^3$$

$$d = 7.762\left(10^{-3}\right)\ \text{ft}$$

$$= 0.0931\ \text{in.} \qquad \textbf{Ans.}$$

Ans:
$d = 0.0931$ in.

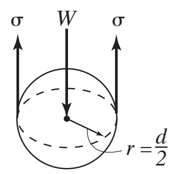
1–59. Sand grains fall a short distance into a tank of water. Determine the largest average diameter of a grain that will float on the water with a contact angle of 180°. Take $\gamma_s = 180\ \text{lb/ft}^3$ and $\sigma = 0.00492\ \text{lb/ft}$. Assume each grain has the shape of a sphere, where $V = \frac{4}{3}\pi r^3$.

SOLUTION

The weight of a sand grain is

$$W = \gamma_s V = \left(180\ \text{lb/ft}^3\right)\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right] = \left(30\pi d^3\right)\ \text{lb}$$

Consider the force equilibrium along the vertical by referring to the FBD of the sand grain, Fig. *a*.



(a)

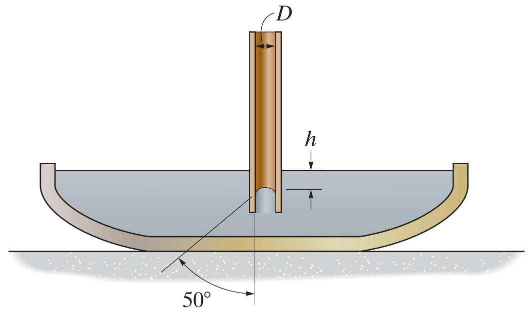
$$+\uparrow \Sigma F_y = 0; \qquad (0.00492\ \text{lb/ft})\left[2\pi\left(\frac{d}{2}\right)\right] - 30\pi d^3 = 0$$

$$d = (0.01281\ \text{ft})\left(\frac{12\ \text{in.}}{\text{ft}}\right)$$

$$= 0.1537\ \text{in.} = 0.154\ \text{in.} \qquad \textbf{Ans.}$$

Ans:
$d = 0.154$ in.

***1–60.** Determine the distance h that the column of mercury in the tube will be depressed when the tube is inserted into the mercury at a room temperature of 68°F. Set $D = 0.12$ in.



SOLUTION

Using the result,

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

From the table in Appendix A, for mercury, $\rho = 26.3\ \text{slug/ft}^3$ and $\sigma = 31.9(10^{-3})\ \dfrac{\text{lb}}{\text{ft}}$.

$$h = \frac{2\left[31.9(10^{-3})\ \dfrac{\text{lb}}{\text{ft}}\right]\cos(180° - 50°)}{\left(26.3\ \dfrac{\text{slug}}{\text{ft}^3}\right)\left(32.2\ \dfrac{\text{ft}}{\text{s}^2}\right)\left[(0.06\ \text{in.})\left(\dfrac{1\ \text{ft}}{12\ \text{in.}}\right)\right]}$$

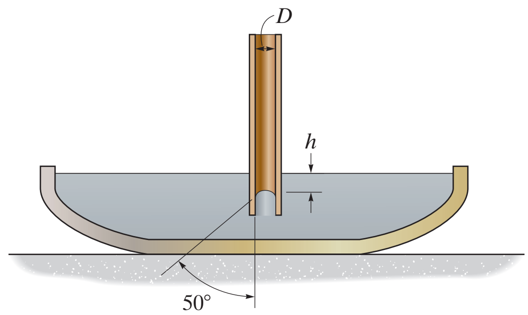
$$= \left[-9.6852(10^{-3})\ \text{ft}\right]\left(\frac{12\ \text{in.}}{1\ \text{ft}}\right)$$

$$= -0.116\ \text{in.} \qquad \textbf{Ans.}$$

The negative sign indicates that a depression occurs.

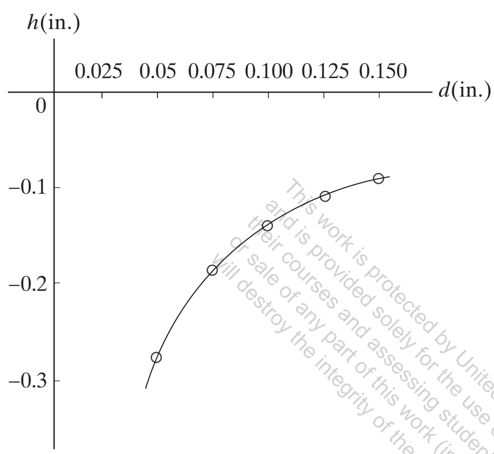
Ans:
$h = -0.116$ in.

1–61. Determine the distance h that the column of mercury in the tube will be depressed when the tube is inserted into the mercury at a room temperature of 68°F. Plot this relationship of h (vertical axis) versus D for $0.05 \text{ in.} \le D \le 0.150 \text{ in.}$ Give values for increments of $\Delta D = 0.025$ in. Discuss this result.



SOLUTION

d(in.)	0.05	0.075	0.100	0.125	0.150
h(in.)	−0.279	−0.186	−0.139	−0.112	0.0930



From the table in Appendix A, for mercury at 68°F, $\rho = 26.3 \text{ slug/ft}^3$, and $\sigma = 31.9(10^{-3})$ lb/ft. Using the result,

$$h = \frac{2\sigma \cos\theta}{\rho g r}$$

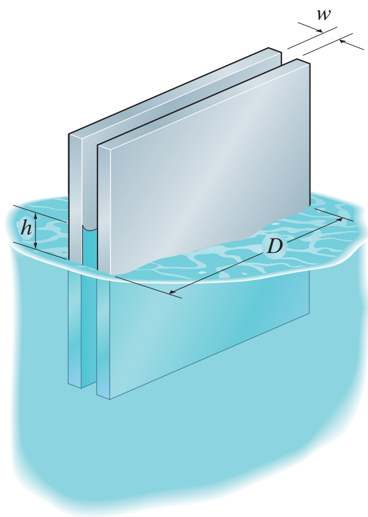
$$h = \left[\frac{2\left[31.9(10^{-3}) \text{ lb/ft}\right] \cos(180° - 50°)}{(26.3 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)[(d/2)(1 \text{ ft}/12 \text{ in.})]}\right]\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)$$

$$h = \left(\frac{-0.01395}{d}\right) \text{in.} \qquad \text{where } d \text{ is in in.}$$

The negative sign indicates that a depression occurs.

Ans:
For $d = 0.075$ in., $h = 0.186$ in.

1–62. Water is at a temperature of 30°C. Plot the height h of the water as a function of the gap w between the two glass plates for $0.4\text{ mm} \leq w \leq 2.4\text{ mm}$. Use increments of 0.4 mm. Take $\sigma = 0.0718\text{ N/m}$.



SOLUTION

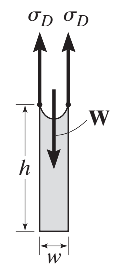
When water contacts the glass wall, $\theta = 0°$. The weight of the rising column of water is

$$W = \gamma_w \forall = \rho_w g(hwD) = \rho_w ghwD$$

Consider the force equilibrium by referring to the FBD of the water column, Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \quad 2(\sigma D) - \rho_w ghwD = 0$$

$$h = \frac{2\sigma}{\rho_w g w}$$



(a)

From the table in Appendix A, $\rho_w = 995.7\text{ kg/m}^3$ at $T = 30°\text{C}$. Then

$$h = \left[\frac{2(0.0718\text{ N/m})}{(995.7\text{ kg/m}^3)(9.81\text{ m/s}^2)[w(10^{-3})\text{ m}]}\right]\left(\frac{10^3\text{ mm}}{1\text{ m}}\right)$$

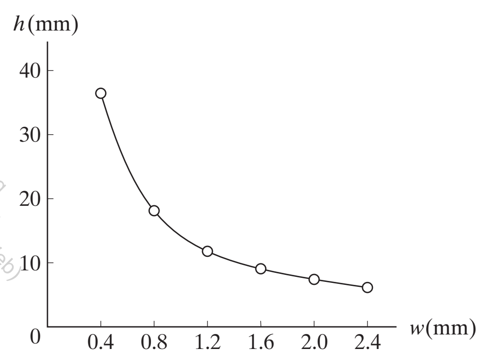
$$= \left(\frac{14.7}{w}\right)\text{mm} \qquad \text{where } w \text{ is in mm.}$$

Ans.

For $0.4\text{ mm} < w < 2.4\text{ mm}$

w (mm)	0.4	0.8	1.2	1.6	2.0	2.4
h (mm)	36.8	18.4	12.3	9.19	7.35	6.13

The plot of h vs. w is shown in Fig. *b*.

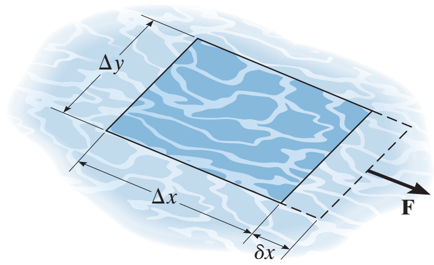


(b)

Ans:

$h = \left(\frac{14.7}{w}\right)\text{mm} \qquad \text{where } w \text{ is in mm.}$

1–63. Because cohesion resists any increase in the surface area of a liquid, it actually tries to *minimize* the size of the surface. Separating the molecules and thus breaking the surface tension requires work, and the energy provided by this work is called ***free-surface energy***. To show how it is related to surface tension, consider the small element of the liquid surface subjected to the surface tension force **F**. If the surface stretches δx, show that the work done by **F** per increase in area is equal to the surface tension in the liquid.



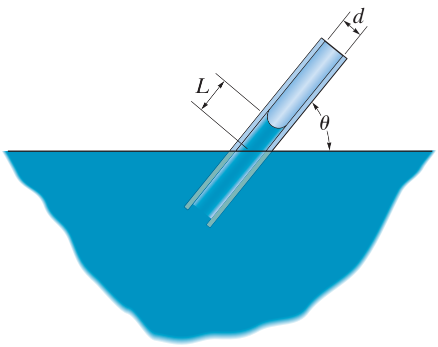
SOLUTION

$$F = \sigma \Delta y$$

$$\text{work} = F\delta x$$

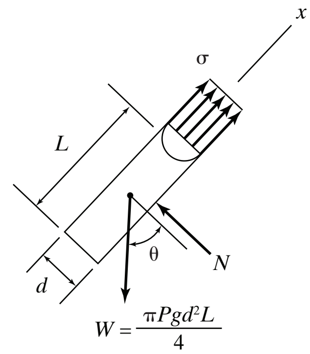
$$\text{work/area increase} = \frac{\sigma \Delta y \delta x}{\Delta y \delta x} = \sigma \quad \textbf{(Q.E.D)}$$

***1–64.** The glass tube has an inner diameter d and is immersed in water at an angle θ from the horizontal. Determine the average length L to which water will rise along the tube due to capillary action. The surface tension of the water is σ and its density is ρ.



SOLUTION

The free-body diagram of the water column is shown in Fig. *a*. The weight of this column is $W = \rho g V = \rho g\left[\pi\left(\frac{d}{2}\right)^2 L\right] = \frac{\pi\rho g d^2 L}{4}$.



(a)

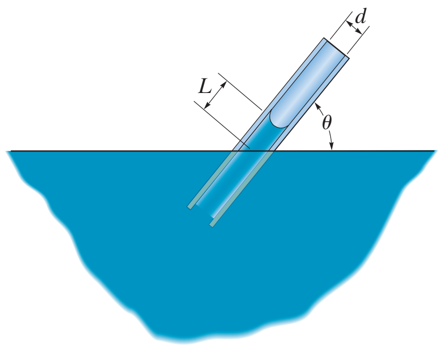
For water, its surface will be almost parallel to the surface of the tube (contact angle $\approx 0°$). Thus, σ acts along the tube. Considering equilibrium along the x axis,

$$\Sigma F_x = 0; \qquad \sigma(\pi d) - \frac{\pi\rho g d^2 L}{4}\sin\theta = 0$$

$$L = \frac{4\sigma}{\rho g d \sin\theta} \qquad \textbf{Ans.}$$

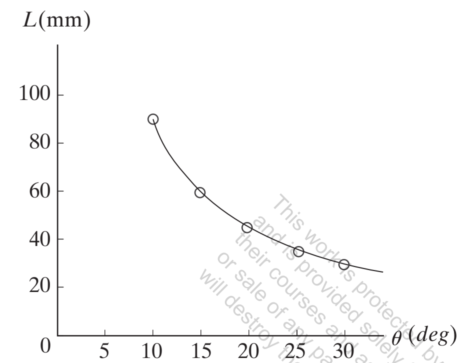
Ans:
$L = 4\sigma/(\rho g d \sin\theta)$

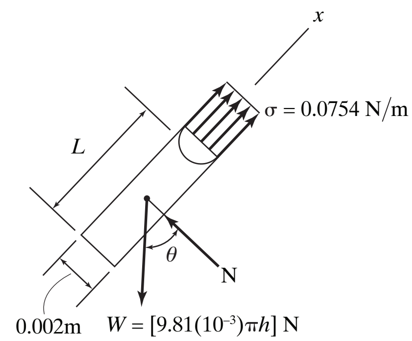
1–65. The glass tube has an inner diameter of $d = 2$ mm and is immersed in water. Determine the average length L to which the water will rise along the tube due to capillary action as a function of the angle of tilt, θ. Plot this relationship of L (vertical axis) versus θ for $10° \leq \theta \leq 30°$. Give values for increments of $\Delta\theta = 5°$. The surface tension of the water is $\sigma = 75.4$ mN/m, and its density is $\rho = 1000$ kg/m^3.



SOLUTION

θ(deg.)	10	15	20	25	30
L(mm)	88.5	59.4	44.9	36.4	30.7





(a)

The FBD of the water column is shown in Fig. *a*. The weight of this column is

$$W = \rho g V = (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left[\frac{\pi}{4}(0.002\ \text{m})L\right] = \left[9.81(10^{-3})\pi L\right]\ \text{N}.$$

For water, its surface will be almost parallel to the surface of the tube ($\theta \cong 0°$) at the point of contact. Thus, σ acts along the tube. Considering equilibrium along x axis,

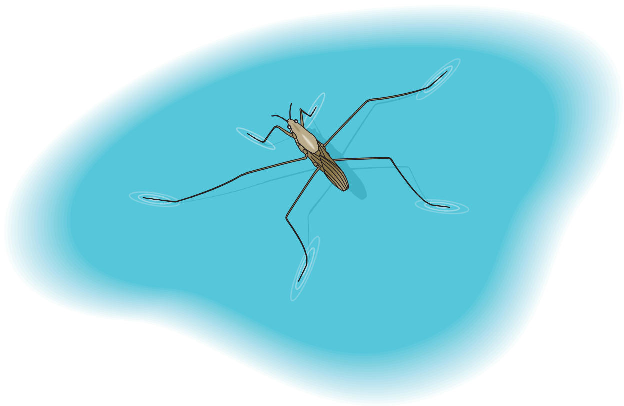
$$\Sigma F_x = 0; \qquad (0.0754\ \text{N/m})\left[\pi(0.002\ \text{m})\right] - \left[9.81(10^{-3})\pi L\right]\sin\theta = 0$$

$$L = \left(\frac{0.0154}{\sin\theta}\right)\text{m} \qquad \text{where } \theta \text{ is in deg.} \qquad \textbf{Ans.}$$

The plot of L versus θ is shown in Fig. *a*.

Ans:
$L = (0.0154/\sin\theta)$ m

1–66. The marine water strider, *Halobates*, has a mass of 0.36 g. If it has six slender legs, determine the minimum contact length of all of its legs combined to support itself in water having a temperature of $T = 20°\text{C}$. Take $\sigma = 72.7\ \text{mN/m}$, and assume the legs to be thin cylinders with a contact angle of $\theta = 0°$.



SOLUTION

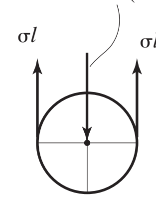
The force supported by the legs is

$$P = \left[0.36\left(10^{-3}\right)\text{kg}\right]\left[9.81\ \text{m/s}^2\right] = 3.5316\left(10^{-3}\right)\text{N}$$

Here, σ is most effective in supporting the weight if it acts vertically upward. This requirement is indicated on the FBD of each leg in Fig. *a*. The force equilibrium along vertical requires

$$+\uparrow \Sigma F_y = 0; \qquad 3.5316\left(10^{-3}\right)\text{N} - 2(0.0727\ \text{N/m})l = 0$$

$$l = 24.3\left(10^{-3}\right)\text{m} = 24.3\ \text{mm} \qquad \textbf{Ans.}$$

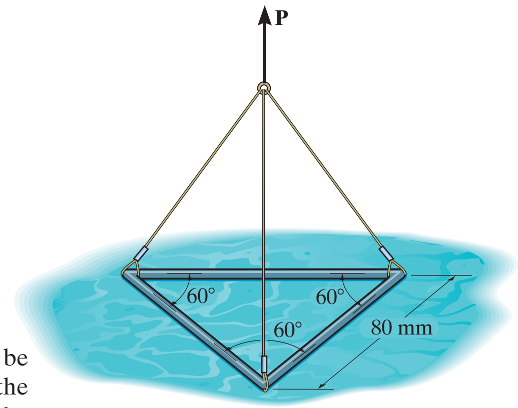


(a)

Note: Because of surface microstructure, a water strider's legs are highly hydrophobic. That is why the water surface curves *downward* with $\theta \approx 0°$, instead of upward as it does when water meets glass.

Ans:
$l = 24.3\ \text{mm}$

1–67. The triangular glass rod has a weight of 0.3 N and is suspended on the surface of the water, for which $\sigma = 0.0728$ N/m. Determine the vertical force **P** needed to pull the rod free from the surface.



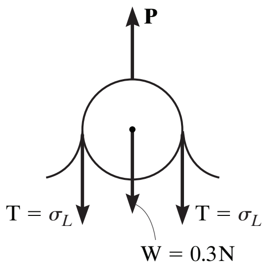
SOLUTION

The free-body diagram of the rod is shown in Fig. *a*. For water, its surface will be almost parallel to the surface of the rod ($\theta \approx 0°$) at the point of contact. When the rod is on the verge of being pulled free, consider the force equilibrium along the vertical.

$$+\uparrow \Sigma F_y = 0; \qquad P - W - 2T = 0$$

$$P - 0.3\text{ N} - 2\{(0.0728\text{ N/m})[3(0.08\text{ m})]\} = 0$$

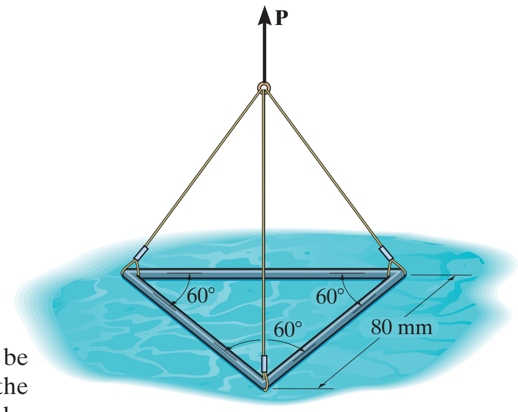
$$P = 0.3349\text{ N} = 0.335\text{ N}$$ **Ans.**



(a)

Ans:
$P = 0.335$ N

***1–68.** The triangular glass rod has a weight of 0.3 N and is suspended on the surface of the water. If it takes a force of $P = 0.335$ N to lift it free from the surface, determine the surface tension of the water.

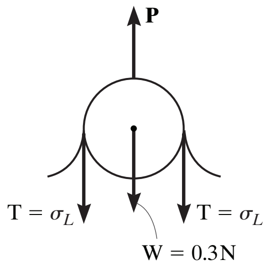


SOLUTION

The free-body diagram of the rod is shown in Fig. *a*. For water, its surface will be almost parallel to the surface of the rod ($\theta \simeq 0°$) at the point of contact. When the rod is on the verge of being pulled free, consider the force equilibrium along the vertical.

$$+\uparrow \Sigma F_y = 0; \quad 0.335\text{ N} - 0.3\text{ N} - 2\{\sigma[3(0.08\text{ m})]\} = 0$$

$$\sigma = 0.07292\text{ N/m} = 0.0729\text{ N/m} \qquad \textbf{Ans.}$$



(a)

Ans:
$\sigma = 0.0729$ N/m