Exam

Name___________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the system of equations.

1) $x_1 - x_2 + 3x_3 = -8$ 1) ________
$2x_1 + x_3 = 0$
$x_1 + 5x_2 + x_3 = 40$

A) (8, 8, 0) B) (-8, 0, 0) C) (0, -8, -8) D) (0, 8, 0)

2) $x_1 + 3x_2 + 2x_3 = 11$ 2) ________
$4x_2 + 9x_3 = -12$
$x_3 = -4$

A) (-4, 1, 6) B) (1, 6, -4) C) (1, -4, 6) D) (-4, 6, 1)

3) $x_1 - x_2 + 8x_3 = -107$ 3) ________
$6x_1 + x_3 = 17$
$3x_2 - 5x_3 = 89$

A) (5, -8, -13) B) (5, 8, -13) C) (-5, -8, 13) D) (-5, 8, 13)

4) $4x_1 - x_2 + 3x_3 = 12$ 4) ________
$2x_1 + 9x_3 = -5$
$x_1 + 4x_2 + 6x_3 = -32$

A) (2, 7, 1) B) (2, -7, 1) C) (2, -7, -1) D) (2, 7, -1)

5) $x_1 + x_2 + x_3 = 6$ 5) ________
$x_1 - x_3 = -2$
$x_2 + 3x_3 = 11$

A) (1, 2, 3) B) (-1, 2, -3) C) No solution D) (0, 1, 2)

6) $x_1 + x_2 + x_3 = 7$ 6) ________
$x_1 - x_2 + 2x_3 = 7$
$5x_1 + x_2 + x_3 = 11$

A) (1, 2, 4) B) (4, 2, 1) C) (4, 1, 2) D) (1, 4, 2)

7) $x_1 - x_2 + x_3 = 8$ 7) ________
$x_1 + x_2 + x_3 = 6$
$x_1 + x_2 - x_3 = -12$

A) (-2, -1, 9) B) (-2, -1, -9) C) (2, -1, 9) D) (2, -1, -9)

8) $5x_1 + 2x_2 + x_3 = -11$ 8) ________
$2x_1 - 3x_2 - x_3 = 17$
$7x_1 + x_2 + 2x_3 = -4$

A) (-3, 0, 4) B) (0, -6, 1) C) (3, 0, -4) D) (0, 6, -1)

9) $7x_1 + 7x_2 + x_3 = 1$ 9) _______
$x_1 + 8x_2 + 8x_3 = 8$
$9x_1 + x_2 + 9x_3 = 9$
A) (-1, 1, 1) B) (1, -1, 1) C) (0, 0, 1) D) (0, 1, 0)

10) $2x_1 + x_2 \quad = 0$ 10) _______
$x_1 - 3x_2 + x_3 = 0$
$3x_1 + x_2 - x_3 = 0$
A) (1, 0, 0) B) No solution C) (0, 1, 0) D) (0, 0, 0)

Determine whether the system is consistent.

11) $x_1 + x_2 + x_3 = 7$ 11) _______
$x_1 - x_2 + 2x_3 = 7$
$5x_1 + x_2 + x_3 = 11$
A) No B) Yes

12) $5x_1 + 2x_2 + x_3 = -11$ 12) _______
$2x_1 - 3x_2 - x_3 = 17$
$7x_1 + x_2 + 2x_3 = -4$
A) No B) Yes

13) $4x_1 - x_2 + 3x_3 = 12$ 13) _______
$2x_1 \quad + 9x_3 = -5$
$x_1 + 4x_2 + 6x_3 = -32$
A) Yes B) No

14) $2x_1 + x_2 \quad = 0$ 14) _______
$x_1 - 3x_2 + x_3 = 0$
$3x_1 + x_2 - x_3 = 0$
A) No B) Yes

15) $x_1 + x_2 + x_3 = 6$ 15) _______
$x_1 \quad - x_3 = -2$
$x_2 + 3x_3 = 11$
A) No B) Yes

16) $x_1 - x_2 + 3x_3 = -11$ 16) _______
$-4x_1 + 4x_2 - 12x_3 = -2$
$x_1 + 3x_2 \quad + x_3 = -17$
A) No B) Yes

17) $x_1 + x_2 + x_3 = 7$ 17) _______
$x_1 - x_2 + 2x_3 = 7$
$2x_1 \quad + 3x_3 = 15$
A) No B) Yes

18) $x_1 + 3x_2 + 2x_3 = 11$
$4x_2 + 9x_3 = -12$
$x_1 + 7x_2 + 11x_3 = -11$ 18) ______

A) Yes B) No

19) $5x_1 + 2x_2 + x_3 = -11$
$2x_1 - 3x_2 - x_3 = 17$
$7x_1 - x_2 = 12$ 19) ______

A) Yes B) No

20) $5x_2 + x_4 = -21$
$x_1 + x_2 + 4x_3 - x_4 = 4$
$5x_1 + x_3 + 4x_4 = 12$
$x_1 + x_2 + 6x_3 = 5$ 20) ______

A) Yes B) No

Determine whether the matrix is in echelon form, reduced echelon form, or neither.

21) $\begin{bmatrix} 1 & 3 & 5 & -7 \\ 0 & 1 & -4 & -4 \\ 0 & 0 & 1 & 6 \end{bmatrix}$ 21) ______

A) Reduced echelon form B) Neither C) Echelon form

22) $\begin{bmatrix} 1 & 4 & 5 & -7 \\ 0 & 1 & -4 & -5 \\ 0 & 6 & 1 & 4 \end{bmatrix}$ 22) ______

A) Echelon form B) Neither C) Reduced echelon form

23) $\begin{bmatrix} 1 & 4 & 5 & -7 \\ 3 & 1 & -4 & -6 \\ 0 & 4 & 1 & 3 \end{bmatrix}$ 23) ______

A) Neither B) Echelon form C) Reduced echelon form

24) $\begin{bmatrix} 1 & 0 & 0 & -7 \\ 1 & 1 & 0 & 1 \\ 0 & 3 & 1 & 1 \end{bmatrix}$ 24) ______

A) Reduced echelon form B) Neither C) Echelon form

25) $\begin{bmatrix} 1 & 4 & 1 & -7 \\ 0 & 1 & -4 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 25) ______

A) Echelon form B) Neither C) Reduced echelon form

26) $\begin{bmatrix} 1 & 0 & 5 & -4 \\ 0 & 1 & -5 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 26) ______

A) Neither B) Reduced echelon form C) Echelon form

27) $\begin{bmatrix} 1 & -5 & -5 & -5 \\ 0 & 0 & -2 & 3 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 27) ______

A) Reduced echelon form B) Neither C) Echelon form

Use the row reduction algorithm to transform the matrix into echelon form or reduced echelon form as indicated.

28) Find the echelon form of the given matrix. 28) ______

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ -3 & -11 & 9 & -5 \\ -2 & 4 & -3 & 4 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 12 & -7 & 10 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & -43 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & -43 & -38 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & -19 & -2 \end{bmatrix}$

29) Find the reduced echelon form of the given matrix. 29) ______

$$\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 2 & 5 & -4 & -1 & 4 \\ -3 & -9 & 9 & 2 & 2 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & 0 & 3 & 0 & 14 \\ 0 & 1 & -2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 0 & 0 & 14 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 4 & -5 & 0 & -2 \\ 0 & 1 & -2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

The augmented matrix is given for a system of equations. If the system is consistent, find the general solution. Otherwise state that there is no solution.

30) $\begin{bmatrix} 1 & -5 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ 30) ______

A) $x_1 = -1 + 5x_2$
$x_2 = 3$
x_3 is free

B) $x_1 = -1 + 5x_2$
x_2 is free

C) No solution

D) (-1, 3)

31) $\begin{bmatrix} 1 & 2 & -3 & -9 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 31) _______

A) $x_1 = -19 + 11x_3$
$x_2 = 5 - 4x_3$
$x_3 = 1$

B) $x_1 = -9 - 2x_2 + 3x_3$
x_2 is free
x_3 is free

C) $x_1 = -19 + 11x_3$
$x_2 = 5 - 4x_3$
x_3 is free

D) No solution

32) $\begin{bmatrix} 1 & 2 & -3 & 6 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 32) _______

A) $x_1 = 6 - 2x_2 + 3x_3$
$x_2 = -7 - 4x_3$
x_3 is free

B) $x_1 = 6 - 2x_2 + 3x_3$
x_2 is free
x_3 is free

C) $x_1 = 20 + 11x_3$
$x_2 = -7 - 4x_3$
$x_3 = 0$

D) $x_1 = 20 + 11x_3$
$x_2 = -7 - 4x_3$
x_3 is free

33) $\begin{bmatrix} 1 & 0 & 6 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ 33) _______

A) $x_1 = 2 - 6x_3$
x_2 is free
$x_3 = \frac{3}{2} + \frac{1}{2}x_2$

B) No solution

C) $x_1 = 2 - 6x_3$
$x_2 = -3 + 2x_3$
$x_3 = 0$

D) $x_1 = 2 - 6x_3$
$x_2 = -3 + 2x_3$
x_3 is free

34) $\begin{bmatrix} 1 & 4 & -2 & -3 & 1 \\ 0 & 0 & 1 & 4 & -4 \\ -1 & -4 & -1 & -9 & 11 \end{bmatrix}$ 34) _______

A) $x_1 = -7 - 4x_2 - 5x_4$
x_2 is free
$x_3 = -4 - 4x_4$
$x_4 = 0$

B) $x_1 = -7 - 4x_2 - 5x_4$
x_2 is free
$x_3 = -4 - 4x_4$
x_4 is free

C) $x_1 = -7 - 4x_2 - 5x_3$
$x_2 = -4 - 4x_3$
x_3 is free

D) $x_1 = -4x_2 + 2x_3 + 3x_4 + 1$
x_2 is free
$x_3 = -4 - 4x_4$
x_4 is free

35) $\begin{bmatrix} 1 & 6 & 3 & -1 & 2 & 6 \\ 0 & 0 & 0 & -4 & 3 & 4 \\ 0 & 0 & 0 & 0 & -2 & 8 \end{bmatrix}$ 35) ______

A) $x_1 = -6x_2 - 3x_3 + 10$
x_2 is free
$x_3 = -4$
$x_4 = \frac{3}{4}x_5 - 1$
$x_5 = -4$

B) No solution

C) $x_1 = -6x_2 - 3x_3 + 10$
x_2 is free
x_3 is free
$x_4 = -4$
$x_5 = -4$

D) $x_1 = -6x_2 - 3x_3 + x_4 - 2x_5 + 6$
x_2 is free
x_3 is free
$x_4 = \frac{3}{4}x_5 - 1$
$x_5 = -4$

Find the indicated vector.

36) Let u = $\begin{bmatrix} -6 \\ 4 \end{bmatrix}$, v = $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Find u + v. 36) ______

A) $\begin{bmatrix} -8 \\ 3 \end{bmatrix}$ B) $\begin{bmatrix} -5 \\ 6 \end{bmatrix}$ C) $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$ D) $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

37) Let u = $\begin{bmatrix} 9 \\ -8 \end{bmatrix}$, v = $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$. Find u - v. 37) ______

A) $\begin{bmatrix} 17 \\ 5 \end{bmatrix}$ B) $\begin{bmatrix} 2 \\ -10 \end{bmatrix}$ C) $\begin{bmatrix} 7 \\ -15 \end{bmatrix}$ D) $\begin{bmatrix} 16 \\ -6 \end{bmatrix}$

38) Let u = $\begin{bmatrix} -5 \\ 5 \end{bmatrix}$, v = $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$. Find v - u. 38) ______

A) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$ B) $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} 11 \\ -4 \end{bmatrix}$ D) $\begin{bmatrix} -4 \\ 11 \end{bmatrix}$

39) Let u = $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Find 4u. 39) ______

A) $\begin{bmatrix} 12 \\ -8 \end{bmatrix}$ B) $\begin{bmatrix} 12 \\ 8 \end{bmatrix}$ C) $\begin{bmatrix} -12 \\ 8 \end{bmatrix}$ D) $\begin{bmatrix} -12 \\ -8 \end{bmatrix}$

40) Let u = $\begin{bmatrix} 8 \\ -6 \end{bmatrix}$. Find 3u. 40) ______

A) $\begin{bmatrix} -24 \\ 18 \end{bmatrix}$ B) $\begin{bmatrix} 24 \\ -18 \end{bmatrix}$ C) $\begin{bmatrix} 24 \\ 18 \end{bmatrix}$ D) $\begin{bmatrix} -24 \\ -18 \end{bmatrix}$

41) Let u = $\begin{bmatrix} -6 \\ -7 \end{bmatrix}$. Find -2u. 41) ______

A) $\begin{bmatrix} 12 \\ -14 \end{bmatrix}$
B) $\begin{bmatrix} -12 \\ 14 \end{bmatrix}$
C) $\begin{bmatrix} -12 \\ -14 \end{bmatrix}$
D) $\begin{bmatrix} 12 \\ 14 \end{bmatrix}$

42) Let u = $\begin{bmatrix} -7 \\ 5 \end{bmatrix}$. Find -5u. 42) ______

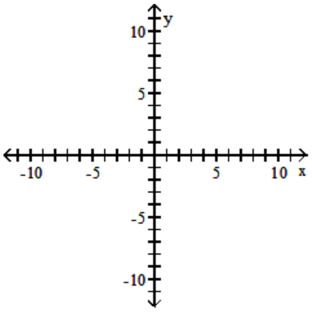
A) $\begin{bmatrix} -35 \\ 25 \end{bmatrix}$
B) $\begin{bmatrix} 35 \\ -25 \end{bmatrix}$
C) $\begin{bmatrix} -35 \\ -25 \end{bmatrix}$
D) $\begin{bmatrix} 35 \\ 25 \end{bmatrix}$

43) Let u = $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, v = $\begin{bmatrix} 7 \\ -5 \end{bmatrix}$. Find -5u + 2v. 43) ______

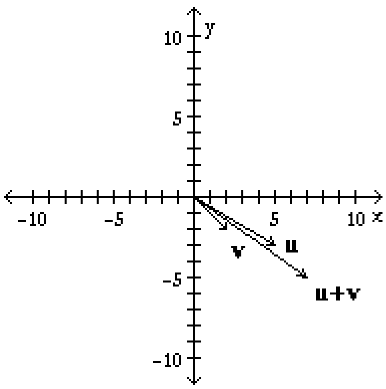
A) $\begin{bmatrix} -15 \\ 4 \end{bmatrix}$
B) $\begin{bmatrix} -24 \\ 5 \end{bmatrix}$
C) $\begin{bmatrix} 4 \\ -15 \end{bmatrix}$
D) $\begin{bmatrix} -45 \\ -8 \end{bmatrix}$

Display the indicated vector(s) on an xy-graph.

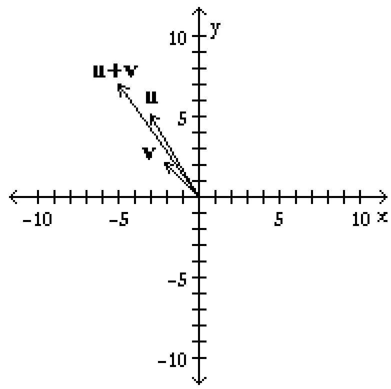
44) Let u = $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ and v = $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Display the vectors u, v, and u + v on the same axes. 44) ______



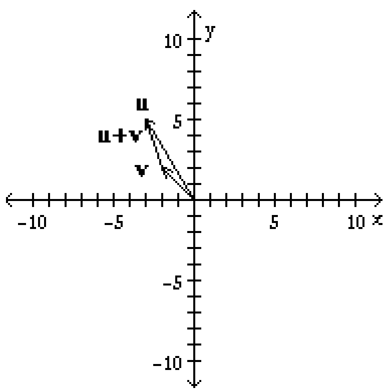
A)



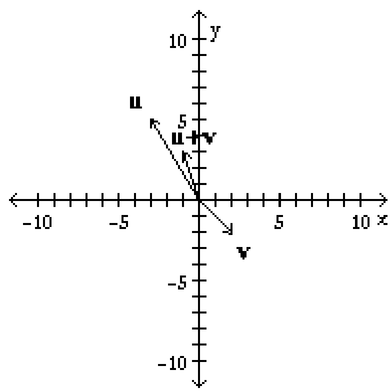
B)



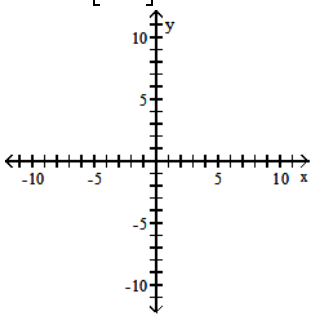
C)



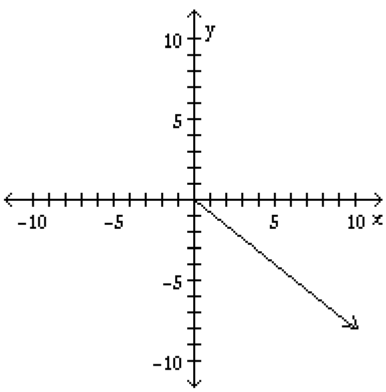
D)



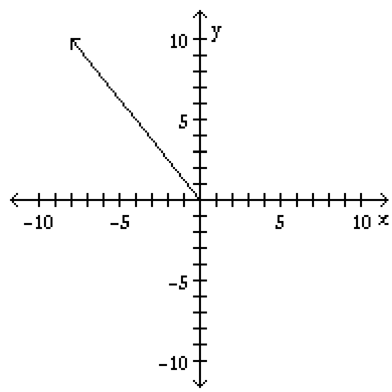
45) Let $u = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$ Display the vector $2u$ using the given axes. 45) ______



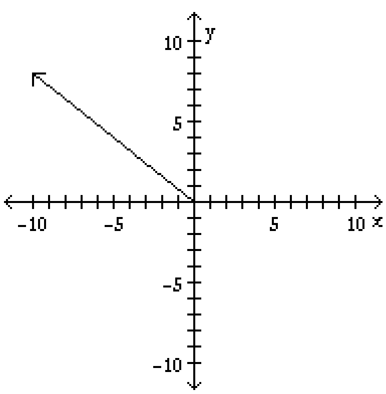
A)



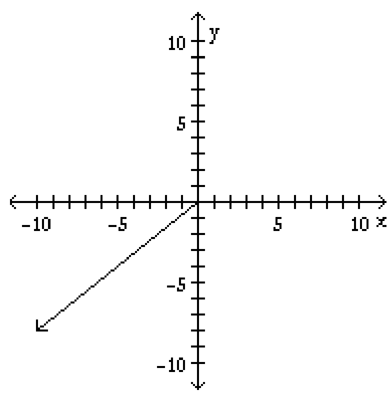
B)



C)



D)



Solve the problem.

46) Let $a_1 = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix}$, $a_2 = \begin{bmatrix} -5 \\ 1 \\ 6 \end{bmatrix}$, and $b = \begin{bmatrix} -39 \\ -1 \\ 22 \end{bmatrix}$. 46) ______

Determine whether b can be written as a linear combination of a_1 and a_2. In other words, determine whether weights x_1 and x_2 exist, such that $x_1 a_1 + x_2 a_2 = b$. Determine the weights x_1 and x_2 if possible.

A) $x_1 = -3, x_2 = 2$ B) No solution C) $x_1 = -4, x_2 = 4$ D) $x_1 = -4, x_2 = 3$

47) Let $a_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $a_2 = \begin{bmatrix} -3 \\ -4 \\ 1 \end{bmatrix}$, $a_3 = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$, and $b = \begin{bmatrix} -3 \\ 6 \\ -1 \end{bmatrix}$. 47) ______

Determine whether b can be written as a linear combination of a_1, a_2, and a_3. In other words, determine whether weights x_1, x_2, and x_3 exist, such that $x_1 a_1 + x_2 a_2 + x_3 a_3 = b$. Determine the weights x_1, x_2, and x_3 if possible.

A) $x_1 = -5$, $x_2 = 0$, $x_3 = 1$

B) $x_1 = -2$, $x_2 = -1$, $x_3 = 6$

C) No solution

D) $x_1 = 2$, $x_2 = 1$, $x_3 = -1$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

48) A company manufactures two products. For $1.00 worth of product A, the company spends $0.50 on materials, $0.20 on labor, and $0.10 on overhead. For $1.00 worth of product B, the company spends $0.40 on materials, $0.20 on labor, and $0.10 on overhead. Let 48) ______________

$$a = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.10 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0.40 \\ 0.20 \\ 0.10 \end{bmatrix}.$$

Then a and b represent the "costs per dollar of income" for the two products.
Evaluate 500a + 100b and give an economic interpretation of the result.

49) A company manufactures two products. For $1.00 worth of product A, the company spends $0.45 on materials, $0.20 on labor, and $0.10 on overhead. For $1.00 worth of product B, the company spends $0.40 on materials, $0.20 on labor, and $0.10 on overhead. Let 49) ______________

$$a = \begin{bmatrix} 0.45 \\ 0.20 \\ 0.10 \end{bmatrix} \text{ and } b = \begin{bmatrix} 0.40 \\ 0.20 \\ 0.10 \end{bmatrix}.$$

Then a and b represent the "costs per dollar of income" for the two products.
Suppose the company manufactures x_1 dollars worth of product A and x_2 dollars worth of product B and that its total costs for materials are $205, its total costs for labor are $100, and its total costs for overhead are $50.
Determine x_1 and x_2, the dollars worth of each product produced. Include a vector equation as part of your solution.

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Compute the product or state that it is undefined.

50) $\begin{bmatrix} -7 & 2 & 7 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ -3 \end{bmatrix}$ 50) ______

A) $[435]$

B) $[-84]$

C) $\begin{bmatrix} -63 \\ 0 \\ -21 \end{bmatrix}$

D) $\begin{bmatrix} -63 & 0 & -21 \end{bmatrix}$

51) $\begin{bmatrix} -1 & 1 & -2 \\ -8 & 2 & -6 \end{bmatrix}\begin{bmatrix} 5 \\ 9 \\ 9 \end{bmatrix}$ 51) ______

A) $\begin{bmatrix} -14 \\ -76 \end{bmatrix}$

B) $\begin{bmatrix} -1 & 1 & -2 \\ -8 & 2 & -6 \\ 5 & 9 & 9 \end{bmatrix}$

C) $[-14\ \ -76]$

D) $\begin{bmatrix} -76 \\ -14 \end{bmatrix}$

52) 52) ______

$\begin{bmatrix} 3 & -7 \\ 8 & -4 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} -4 \\ 5 \end{bmatrix}$

A) $\begin{bmatrix} -47 \\ -52 \\ 21 \end{bmatrix}$

B) $\begin{bmatrix} -12 & 40 \\ 28 & -20 \\ -4 & 1 \end{bmatrix}$

C) Undefined

D) $\begin{bmatrix} -12 & -35 \\ -32 & -20 \\ 16 & 5 \end{bmatrix}$

53) 53) ______

$\begin{bmatrix} -4 & 8 \\ 1 & 5 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$

A) $\begin{bmatrix} -4 & 8 \\ -5 & -25 \\ 14 & 56 \end{bmatrix}$

B) Undefined

C) $\begin{bmatrix} 4 \\ -30 \\ 70 \end{bmatrix}$

D) $[5\ \ 39]$

Write the system as a vector equation or matrix equation as indicated.

54) Write the following system as a vector equation involving a linear combination of vectors. 54) ______

$5x_1 - 2x_2 - x_3 = 2$

$4x_1 + \qquad 3x_3 = -1$

A) $x_1\begin{bmatrix} 5 \\ 4 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

B) $x_1\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

C) $x_1\begin{bmatrix} 5 \\ 4 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

D) $5\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - 2\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

55) Write the following system as a matrix equation involving the product of a matrix and a vector on th side and a vector on the right side. 55) ______

$3x_1 + \ x_2 - 2x_3 = -4$

$2x_1 - 2x_2 \qquad = 1$

A) $\begin{bmatrix} x_1 & x_2 & x_3 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} 3 & 1 & -2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} 3 & 2 \\ 1 & -2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 1 & -2 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

Solve the problem.

56) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -4 & -7 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. 56) ______

Determine if the equation $Ax = b$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible b_1, b_2, b_3, give a description of the set of all b for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

A) Equation is consistent for all b_1, b_2, b_3 satisfying $-3b_1 + b_3 = 0$.
B) Equation is consistent for all b_1, b_2, b_3 satisfying $2b_1 + b_2 = 0$.
C) Equation is consistent for all possible b_1, b_2, b_3.
D) Equation is consistent for all b_1, b_2, b_3 satisfying $7b_1 + 5b_2 + b_3 = 0$.

57) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -3 & -12 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. 57) ______

Determine if the equation $Ax = b$ is consistent for all possible b_1, b_2, b_3. If the equation is not consistent for all possible b_1, b_2, b_3, give a description of the set of all b for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

A) Equation is consistent for all b_1, b_2, b_3 satisfying $9b_1 + 6b_2 + b_3 = 0$.
B) Equation is consistent for all b_1, b_2, b_3 satisfying $-3b_1 + b_3 = 0$.
C) Equation is consistent for all b_1, b_2, b_3 satisfying $-b_1 + b_2 + b_3 = 0$.
D) Equation is consistent for all possible b_1, b_2, b_3.

58) Find the general solution of the simple homogeneous "system" below, which consists of a single linear equation. Give your answer as a linear combination of vectors. Let x_2 and x_3 be free variables 58) ______

$-2x_1 - 8x_2 + 16x_3 = 0$

A)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -8 \\ 0 \\ 1 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

B)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = -4 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - 8 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

C)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 8 \\ 0 \\ 1 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

D)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix} \quad \text{(with } x_2, x_3 \text{ free)}$$

59) Find the general solution of the homogeneous system below. Give your answer as a vector. 59) ______

$$x_1 + 2x_2 - 3x_3 = 0$$
$$4x_1 + 7x_2 - 9x_3 = 0$$
$$-x_1 - 4x_2 + 9x_3 = 0$$

A) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$

D) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3\begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$

60) Describe all solutions of $Ax = b$, where 60) ______

$$A = \begin{bmatrix} 2 & -5 & 3 \\ -2 & 6 & -5 \\ -4 & 7 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} -5 \\ 2 \\ 19 \end{bmatrix}.$$

Describe the general solution in parametric vector form.

A) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ -3 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 7/2 \\ 2 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ -3 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} 7/2 \\ 2 \\ 0 \end{bmatrix}$

C) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

D) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 2 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} -10 \\ -3 \\ 0 \end{bmatrix}$

61) Suppose an economy consists of three sectors: Energy (E), Manufacturing (M), and Agriculture (A). 61) ______
Sector E sells 70% of its output to M and 30% to A.
Sector M sells 30% of its output to E, 50% to A, and retains the rest.
Sector A sells 15% of its output to E, 30% to M, and retains the rest.

Denote the prices (dollar values) of the total annual outputs of the Energy, Manufacturing, and Agri
sectors by p_e, p_m, and p_a, respectively. If possible, find equilibrium prices that make each sector's ir
match its expenditures.
Find the general solution as a vector, with p_a free.

A) $\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.308\ p_a \\ 0.716\ p_a \\ p_a \end{bmatrix}$

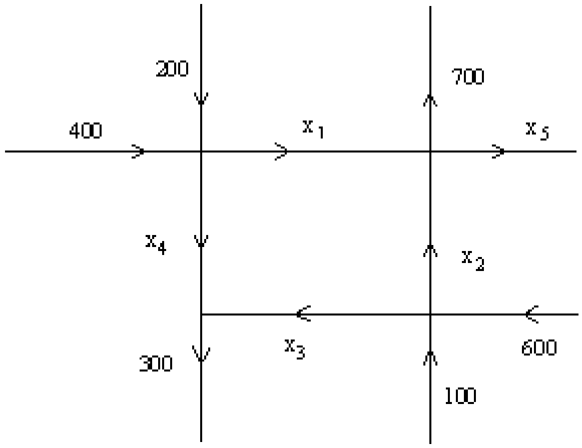
B) $\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.607\ p_a \\ 0.481\ p_a \\ p_a \end{bmatrix}$

C) $\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.465\ p_a \\ 0.593\ p_a \\ p_a \end{bmatrix}$

D) $\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.356\ p_a \\ 0.686\ p_a \\ p_a \end{bmatrix}$

62) The network in the figure shows the traffic flow (in vehicles per hour) over several one-way streets in the downtown area of a certain city during a typical lunch time. Determine the general flow patte the network. 62) ______

In other words, find the general solution of the system of equations that describes the flow. In your general solution let x_4 be free.



A) $x_1 = 600 - x_4$
$x_2 = 400 + x_4$
$x_3 = 300 - x_4$
x_4 is free
$x_5 = 300$

B) $x_1 = 500 + x_4$
$x_2 = 400 - x_4$
$x_3 = 300 - x_4$
x_4 is free
$x_5 = 200$

C) $x_1 = 600 - x_4$
$x_2 = 400 - x_4$
$x_3 = 300 + x_4$
x_4 is free
$x_5 = 300$

D) $x_1 = 600 + x_5$
$x_2 = 400 - x_5$
$x_3 = 300 - x_5$
$x_4 = 300$
x_5 is free

63) Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ -4 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 8 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$. 63) ______

Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent.

A) No
B) Yes

64) Determine if the columns of the matrix $A = \begin{bmatrix} -2 & 1 & 4 \\ 4 & 0 & -4 \\ 2 & 4 & 6 \end{bmatrix}$ are linearly independent. 64) ______

A) No
B) Yes

65) For what values of h are the given vectors linearly independent? 65) ______

$\begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \\ h \end{bmatrix}$

A) Vectors are linearly independent for $h \neq 24$
B) Vectors are linearly independent for all h
C) Vectors are linearly independent for $h = 24$
D) Vectors are linearly dependent for all h

66) For what values of h are the given vectors linearly dependent? 66) ______

$$\begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} -20 \\ 12 \\ h \end{bmatrix}$$

A) Vectors are linearly independent for all h
B) Vectors are linearly dependent for h = -20
C) Vectors are linearly dependent for all h
D) Vectors are linearly dependent for h ≠ -20

67) Let $A = \begin{bmatrix} 2 & 8 & -2 \\ 3 & -5 & -3 \end{bmatrix}$ and $u = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$. 67) ______

Define a transformation T: $\mathbb{R}^3 \to \mathbb{R}^2$ by T(x) = Ax. Find T(u), the image of u under the transformation T.

A) $\begin{bmatrix} 4 & -8 & -2 \\ 6 & 5 & -3 \end{bmatrix}$

B) $\begin{bmatrix} -6 \\ 8 \end{bmatrix}$

C) $\begin{bmatrix} 10 \\ -3 \\ -5 \end{bmatrix}$

D) $\begin{bmatrix} 16 \\ 5 \end{bmatrix}$

68) Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $u = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$ into $\begin{bmatrix} -15 \\ 6 \end{bmatrix}$ and maps $v = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$ into $\begin{bmatrix} 24 \\ -12 \end{bmatrix}$. 68) ______

Use the fact that T is linear to find the image of 3u + v.

A) $\begin{bmatrix} 9 \\ -6 \end{bmatrix}$

B) $\begin{bmatrix} 27 \\ -18 \end{bmatrix}$

C) $\begin{bmatrix} -3 \\ 12 \end{bmatrix}$

D) $\begin{bmatrix} -21 \\ 6 \end{bmatrix}$

69) Let $A = \begin{bmatrix} 1 & -3 & 0 \\ -4 & 0 & 2 \\ 2 & -5 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$. 69) ______

Define a transformation T: $\mathbb{R}^3 \to \mathbb{R}^3$ by T(x) = Ax.
If possible, find a vector x whose image under T is b. Otherwise, state that b is not in the range of the transformation T.

A) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

B) $\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

C) $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

D) b is not in the range of the transformation T.

70) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 4 & -1 \\ 2 & -5 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} -5 \\ 2 \\ -4 \end{bmatrix}$. 70) _______

Define a transformation $T: \mathcal{R}^3 \to \mathcal{R}^3$ by $T(x) = Ax$.

If possible, find a vector x whose image under T is b. Otherwise, state that b is not in the range of th transformation T.

A) $\begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$

B) b is not in the range of the transformation T.

C) $\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

D) $\begin{bmatrix} -10 \\ -5 \\ -5 \end{bmatrix}$

Describe geometrically the effect of the transformation T.

71) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. 71) _______

Define a transformation T by $T(x) = Ax$.

A) Horizontal shear

B) Projection onto x_1-axis

C) Projection onto x_2-axis

D) Vertical shear

72) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. 72) _______

Define a transformation T by $T(x) = Ax$.

A) Vertical shear

B) Horizontal shear

C) Projection onto the x_2x_3-plane

D) Projection onto the x_2-axis

Solve the problem.

73) The columns of $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. 73) _______

Suppose that T is a linear transformation from $\mathcal{R}^3$ into $\mathcal{R}^2$ such that

$$T(e_1) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, T(e_2) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \text{and } T(e_3) = \begin{bmatrix} -4 \\ 1 \end{bmatrix}.$$

Find a formula for the image of an arbitrary $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in $\mathcal{R}^3$.

A) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_2 \\ 2x_1 \end{bmatrix}$

B) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 2x_2 - 4x_3 \\ 2x_1 \\ -2x_1 + x_3 \end{bmatrix}$

C) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 2x_2 - 4x_3 \\ -2x_1 \quad\quad + x_3 \end{bmatrix}$

D) $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_2 \\ 2x_1 \\ 4x_2 + x_3 \end{bmatrix}$

Find the standard matrix of the linear transformation T.

74) T: $\mathcal{R}^2$ -> $\mathcal{R}^2$ rotates points (about the origin) through $\frac{7}{4}\pi$ radians (with counterclockwise rotation for a positive angle). 74) _______

A) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

B) $\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix}$

C) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

D) $\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

75) T: $\mathcal{R}^2$ -> $\mathcal{R}^2$ first performs a vertical shear that maps e_1 into $e_1 + 3e_2$, but leaves the vector e_2 unchanged, then reflects the result through the horizontal x_1-axis. 75) _______

A) $\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$

B) $\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix}$

D) $\begin{bmatrix} -1 & 0 \\ 3 & -1 \end{bmatrix}$

Determine whether the linear transformation T is one-to-one and whether it maps as specified.

76) Let T be the linear transformation whose standard matrix is 76) _______

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & -4 \\ -5 & 5 & -6 \end{bmatrix}.$$

Determine whether the linear transformation T is one-to-one and whether it maps $\mathcal{R}^3$ onto $\mathcal{R}^3$.

A) Not one-to-one; not onto $\mathcal{R}^3$

B) One-to-one; not onto $\mathcal{R}^3$

C) One-to-one; onto $\mathcal{R}^3$

D) Not one-to-one; onto $\mathcal{R}^3$

77) $T(x_1, x_2, x_3) = (-2x_2 - 2x_3, -2x_1 + 9x_2 + 5x_3, -x_1 - 2x_3, 3x_2 + 3x_3)$ 77) _______

Determine whether the linear transformation T is one-to-one and whether it maps $\mathbb{R}^3$ onto $\mathbb{R}^4$.

A) One-to-one; onto $\mathbb{R}^4$
B) Not one-to-one; onto $\mathbb{R}^4$
C) One-to-one; not onto $\mathbb{R}^4$
D) Not one-to-one; not onto $\mathbb{R}^4$

Solve the problem.

78) The table shows the amount (in g) of protein, carbohydrate, and fat supplied by one unit (100 g) of tl different foods. 78) _______

	Food 1	Food 2	Food 3
Protein	15	35	25
Carbohydrate	45	30	50
Fat	6	4	1

Betty would like to prepare a meal using some combination of these three foods. She would like the to contain 15 g of protein, 25 g of carbohydrate, and 3 g of fat. How many units of each food should use so that the meal will contain the desired amounts of protein, carbohydrate, and fat? Round to 3 decimal places.

A) 0.360 units of Food 1, 0.204 units of Food 2, 0.055 units of Food 3
B) 0.302 units of Food 1, 0.238 units of Food 2, 0.085 units of Food 3
C) 0.280 units of Food 1, 0.192 units of Food 2, 0.164 units of Food 3
D) 0.326 units of Food 1, 0.247 units of Food 2, 0.059 units of Food 3

79) The population of a city in 2000 was 400,000 while the population of the suburbs of that city in 2000 was 800,000. 79) _______

Suppose that demographic studies show that each year about 5% of the city's population moves to tl suburbs (and 95% stays in the city), while 4% of the suburban population moves to the city (and 96% remains in the suburbs).

Compute the population of the city and of the suburbs in the year 2002. For simplicity, ignore other influences on the population such as births, deaths, and migration into and out of the city/suburban

A) City: 361,000
Suburbs: 737,280
B) City: 422,920
Suburbs: 777,080
C) City: 412,000
Suburbs: 788,000
D) City: 361,000
Suburbs: 839,000

1) D
2) B
3) B
4) C
5) A
6) A
7) A
8) B
9) C
10) D
11) B
12) B
13) A
14) B
15) B
16) A
17) A
18) B
19) B
20) A
21) C
22) B
23) A
24) B
25) A
26) B
27) C
28) C
29) A
30) C
31) D
32) D
33) D
34) B
35) C
36) C
37) B
38) B
39) B
40) B
41) D
42) B
43) C
44) B
45) A
46) D
47) C

48) $500a + 100b = \begin{bmatrix} 290 \\ 120 \\ 60 \end{bmatrix}$

500a + 100b lists the various costs for producing $500 worth of product A and $100 worth of product B, namely $290 for materials, $120 for labor, and $60 for overhead.

49) $x_1a + x_2b = \begin{bmatrix} 205 \\ 100 \\ 50 \end{bmatrix}$

or

$$x_1\begin{bmatrix} 0.45 \\ 0.20 \\ 0.10 \end{bmatrix} + x_2\begin{bmatrix} 0.40 \\ 0.20 \\ 0.10 \end{bmatrix} = \begin{bmatrix} 205 \\ 100 \\ 50 \end{bmatrix}$$

$x_1 = 100, x_2 = 400$

50) B
51) A
52) A
53) B
54) A
55) D
56) C
57) A
58) C
59) D
60) A
61) D
62) A
63) B
64) A
65) A
66) C
67) B
68) D
69) C
70) B
71) D
72) C
73) C
74) A
75) C
76) C
77) D
78) D
79) B